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CRITICAL FLOW SECTION IN A COLLECTOR CHANNEL

By
James C.Y. Guo, PhD, P.E. Professor
Department of Civil Engineering, University of Colorado at Denver, Denver, Colorado 80204.

Abstract: Using the concept of singular point, the critical flow section in a collector channel can be located by solving the dynamic equation of spatially varied flow and Manning's formula. In addition to the channel length and downstream tailwater, the occurrence of a critical flow section in a spatially varied flow also depends on the combination of channel cross sectional geometry, roughness, slope, and inflow rate. When the critical flow section is necessary to be developed in a collector channel, the two dimensionless parameters, F_q/S_o representing the design capacity, and N/S_o representing the channel roughness, derived in this study can provide general guidance in selecting channel cross sectional parameters. A set of design charts is produced in this study for trapezoidal channels with a side slope of 1V:1H, 0.5V:1H or zeroV:1H.

Key Words: Spatially, Collector, Critical Flow

INTRODUCTION

The flow rate in a spatially varied flow increases or decreases downstream. A typical example is a collector channel which is designed to collect excess water from side-weirs of a storage facility. As it is in most cases of practical interest, a collector channel is designed with no flow at the upstream beginning and then collects the lateral flow at a constant rate per unit length. As the discharge increases downstream in a collector channel, a critical flow may exist when the flow varies from subcritical to supercritical flow. The dynamic equation of spatially varied flow can be solved if a control section is located. Unless the downstream tailwater is known, the critical flow section is otherwise considered to be the control section. Location of such a critical flow section is important to design the channel depth because it varies along the collector channel.

If there exists a critical flow section in a spatially varied flow, Bremen and Hager (1989) recommended that the critical flow section be located by the singular point method. This study found that the existence of a critical flow in a spatially varied flow depends on channel parameters and lateral inflow rates. Without a guideline, the selected channel parameters and inflow rates may result in a supercritical flow in which a critical flow section does not exist. This study presents a systematical methodology by which design parameters can be selected to warrant the existence of critical flow. A set of design charts was also established for determining the critical flow locations and conditions in trapezoidal channels. Engineers may use these design charts as a guidance to select design variables and develop alternatives before water surface profile computations.

CRITICAL FLOW IN A SPATIALLY VARIED FLOW

Without upstream inflow, the discharge in a collector channel is proportional to the uniform and steady lateral inflow and the length of the channel. Therefore, we have

$$Q_x = qx \tag{1}$$

in which Q_x = discharge at the location x in the channel, q = lateral inflow rate per unit length to the channel, and x = distance from the beginning of the channel. When the lateral flow is perpendicular to the collector channel, the dynamic equation for spatially varied flow is expressed by the variation of water depth (Chow 1959) as:

$$\frac{dy}{dx} = \frac{S_o - S_e - 2\frac{D}{x}F_r^2}{1 - F_r^2} \tag{2}$$

in which y = flow depth, S_o = channel slope, S_e = energy line slope, g = gravitational acceleration, A = flow area, and D = hydraulic depth which is the ratio between flow area and its top width, T. The Froude number, Fr, is defined by either the hydraulic depth or top width as:

$$F_r^2 = \frac{Q^2}{gA^2D} = \frac{Q^2T}{gA^3} \tag{3}$$

in which Fr = flow Froude number. Eq 2 can be solved if a tailwater condition is known or a control section is located. This study concentrates on the cases that as the discharge increases in the downstream direction, the flow regime in a spatially varied flow changes from subcritical to supercritical flow. Therefore, the critical flow section can serve as the control section. Applying Eq 2 to the critical flow section with the flow Froude number equal to unity, solutions result by simultaneously solving (Bremen and Hager 1989):

$$1 - F_r^2 = 0 \text{ or } F_r^2 = 1 \tag{4}$$

and

$$S_o - S_c - 2\frac{D_c}{x_c} = 0 \tag{5}$$

The subscript, c, represents the parameters associated with the critical flow section. Bremen and Hager (1989) suggested that Eqs 4 and 5 provide solutions for determining the location of the critical section. With the aide of Eq 4, Eq 3 becomes

$$Q_c^2 = \frac{gA_c^3}{T_c} \tag{6}$$

Substituting Eq 1 into Eq 6, the location of the critical flow section, x_c, can be expressed by

$$x_c = \sqrt{\frac{gA_c^3}{q^2T_c}} \tag{7}$$

The friction slope at the critical flow section can be estimated by Manning's equation as:

$$S_c = \frac{Q_c^2n^2}{2.22A_c^{\frac{10}{3}}P_c^{-\frac{4}{3}}} \tag{8}$$

in which n = Manning's roughness coefficient, and P_c = wetted perimeter at the critical flow section. Substituting Eqs 1, 7 and 8 into Eq 5 yields

$$S_o - \frac{gP_c^{\frac{4}{3}}n^2}{2.22T_cA_c^{\frac{1}{3}}} - 2\frac{q}{\sqrt{gT_cA_c}} = 0 \tag{9}$$

Henderson (1966) derived a similar equation to Eq 9 using Chezy's formula. French (1985) achieved the solutions for wide channels using Henderson's suggestion. In this study, Manning's formula was applied to collector channel designs. Eq 9 was found to be so sensitive that solution does not even exist in certain ranges of design parameters. Without a design guidance, engineers have to experiment each selected design alternative through the numerical method to confirm the existence of a critical flow section. In order to establish design guidance for determining the feasible solutions for Eq 9, both Eqs 7 and 9 are further normalized with the channel width, B as:

$$\frac{N}{S_o}\frac{P_*^{\frac{4}{3}}}{T_*A_*^{\frac{1}{3}}} + 2\frac{F_q}{S_o}\frac{1}{\sqrt{T_*A_*}} = 1 \tag{10}$$

$$X_* = \frac{\sqrt{A_*^3/T_*}}{F_q/S_o} \tag{11}$$

in which

$$N = [\frac{n\sqrt{g}}{1.49B^{\frac{1}{6}}}]^2 \tag{12}$$

$$F_q = \frac{q}{\sqrt{gB^3}} \tag{13}$$

$$T_* = 1 + 2zY_* \tag{14}$$

$$A_* = Y_* + zY_*^2 \tag{15}$$

$$P_* = 1 + 2\sqrt{1+z^2}\,Y_* \tag{16}$$

$$Y_* = \frac{Y_c}{B} \tag{17}$$

$$X_* = \frac{S_o x_c}{B} \tag{18}$$

in which z = channel side slope, and the subscript , *, represents the normalized parameters at the critical flow section. For a rectangular channel, Eq 12 can be further reduced to

$$\frac{N}{S_o}\frac{(1+2Y_*)^{\frac{4}{3}}}{Y_*^{\frac{1}{3}}} + 2\frac{F_q}{S_o}\frac{1}{\sqrt{Y_*}} = 1 \tag{19}$$

A higher ratio of N/S_o implies a rough channel on a mild slope, and a higher ratio of F_q/S_o means that the channel is subject to high lateral inflow. As a part of a spillway system or interception dike system, the collector channel is usually designed to have a narrow cross section. Solutions of Eq 10 developed for trapezoidal channels with a side slope of 1V:1H or 2V:1H are plotted in Figures 1 and 2. Solutions of Eq 19 for rectangular channel sections are plotted in Figure 3. Examining Eq 10, it is noticed that solutions do not exist if either one of the two dimensionless groups exceeds unity. For instance, a critical flow section can be developed in trapezoidal collector channels with a high value of N/S_o from 0.01 to 0.50 and F_q/S_o from 0.18 to 17.45, but not for rectangular collector channels whose N/S_o ranges from 0.005 to 0.05 and F_q/S_o ranges from 0.095 to 0.83.

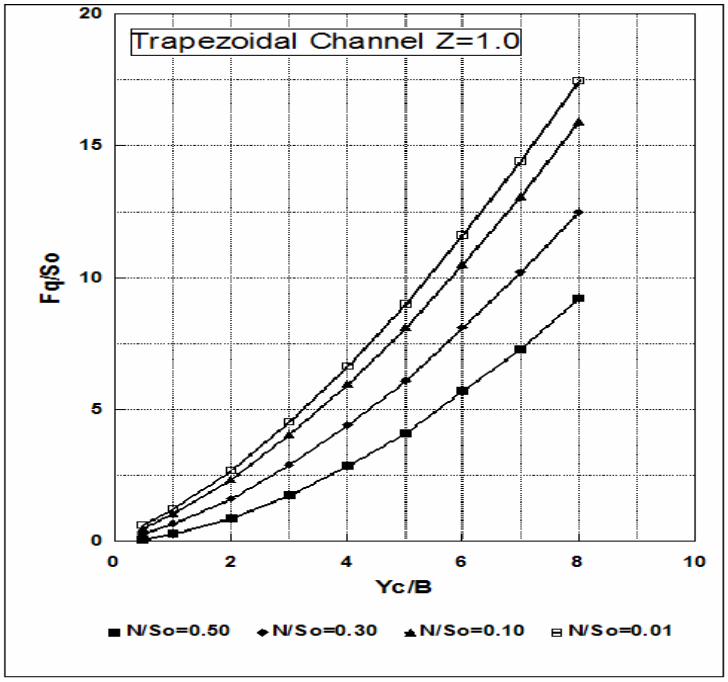

Figure 1. Critical Flow Conditions in a Collector Channel with Side Slope of 1V:1H

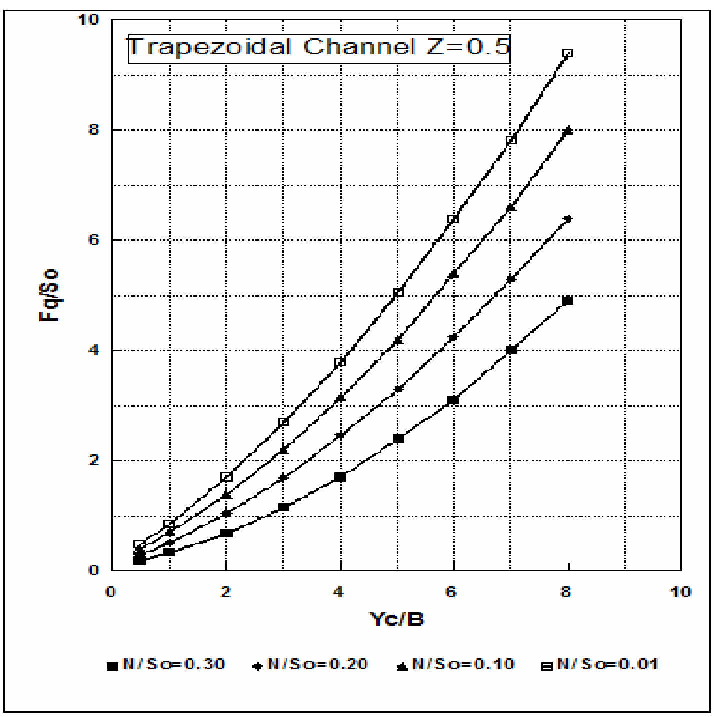

Figure 2. Critical Flow Conditions in a Collector Channel with Side Slope of 0.5V:1H

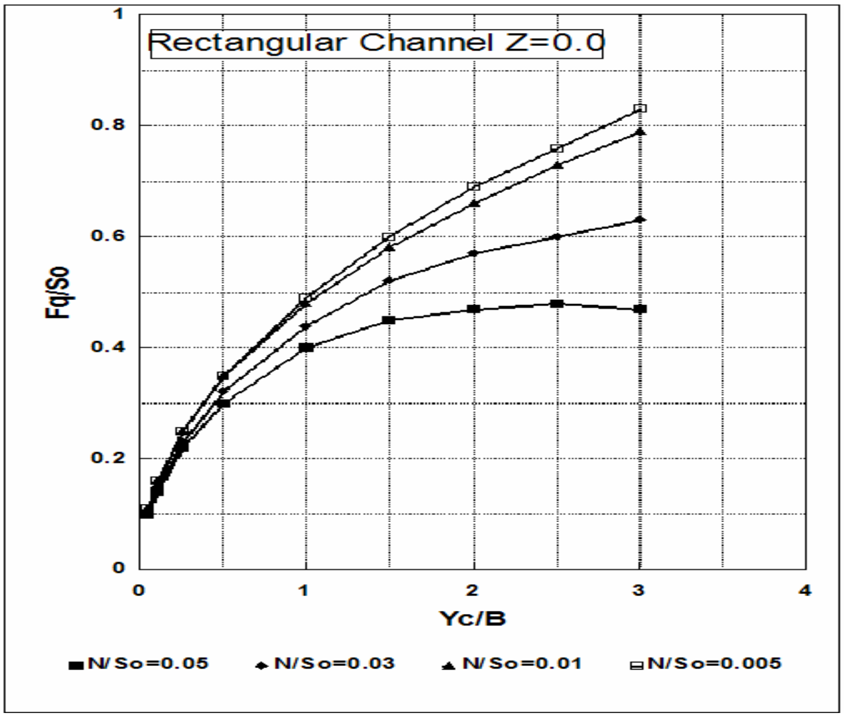

Figure 3. Critical Flow Conditions in a Rectangular Collector Channel

DESIGN PROCEDURES

To explain how to apply the design charts to the design of a collector channel, the example developed by Hinds (1926), illustrated by Chow (1959), and discussed by French (1985) was used. Design charts, Figures 1 to 3, require the following information: q = 40 cfs/ft, n=0.015, S_o = 0.1505 ft/ft, B=10.0 ft, and z = 0.5. The recommended procedures are:

(1) Determination of the Inflow Froude Number, F_q, to Channel Slope Ratio

$$F_q = \frac{q}{\sqrt{gB^3}} = \frac{40}{\sqrt{32.2*10^3}} = 0.223$$

$$\frac{F_q}{S_o} = \frac{0.223}{0.1505} = 1.481$$

(2) Determination of the Roughness Coefficient to Channel Slope Ratio

$$N = \left(\frac{n\sqrt{g}}{1.49B^{\frac{1}{6}}}\right)^2 = \left(\frac{0.015\sqrt{32.2}}{1.49*10.0^{\frac{1}{6}}}\right)^2 = 0.001515$$

$$\frac{N}{S_o} = \frac{0.001515}{0.1515} = 0.01$$

(3) Determination of the Critical Flow Depth

Using Figure 2 with N/S_o=0.01, the ratio, Y_*, is found to be 1.76, or the critical depth for this case is 17.6 ft.

(4) Determination of the Location of the Critical Flow Section using Eq's 12 through 18.

$$A_* = Y_* + zY_*^2 = 1.76 + 0.5 * 1.76^2 = 3.31$$

$$T_* = 1 + 2zY_* = 1 + 2 * 0.5 * 1.76 = 2.76$$

$$D_* = \frac{A_*}{T_*} = \frac{3.31}{2.76} = 1.20$$

$$X_* = \frac{\sqrt{D_* A_*^2}}{F/S_0} = \frac{\sqrt{1.20 * 3.31^2}}{1.481} = 2.448$$

$$x_c = \frac{BX_*}{S_o} = 162.5 \ ft$$

Solutions obtained from the design charts agree with Hinds' graphical solutions. It is interesting to notice that this example has been used for decades to illustrate the water surface profile computations in a spatially varied flow. It is obvious that the channel slope of 0.15015 may serve the mathematical purpose for an illustrative example, but is not easy to be achieved in construction. With the aid of Figures 1 through 3, several alternative channels can also be developed for this case with S_o=0.10, 0.15, or 0.20, and B=10.0 or 15.0 ft. They are summarized in Table 1 for comparison of sensitivity among variables. Figure 1 through 3 provide a systematically approach to identify all possible solutions for Eq's 10 and 19.

Alternative	B (ft)	S_o	F_q/S_o	N/S_o	Z	Y_c/B	S_oX_c/B	Y_c (ft)	X_c (ft)
1.00	10.00	0.15	1.485	0.0100	0.5	1.760	2.44	17.60	162.52
2.00	10.00	0.15	1.485	0.0100	1.0	1.200	1.57	12.00	104.36
3.00	15.00	0.10	1.213	0.0132	1.0	0.995	1.333	6.20	199.95
4.00	15.00	0.20	0.607	0.0066	1.0	0.500	0.757	10.00	56.78
5.00	15.00	0.15	0.808	0.0088	1.0	0.67	0.97	10.10	96.50
6.00	15.00	0.15	0.808	0.0088	0.5	0.94	1.44	14.10	144.20

Table 1. Alternatives Developed for the Case Study of Collector Channel Design

CONCLUSION:

This study expands the singular point method to solve the dynamic equation of spatially varied flow using Manning's formula. It is understood that a critical flow section may not occur if the channel is too short, or drowned by a sufficient tailwater. This study found that the existence of a critical flow also depends on the combination of channel cross sectional geometry, roughness, slope, and inflow rate. When a critical flow section is required for design, care must be taken in selecting channel parameters. To assure the existence of a critical flow section, neither of the two dimensionless parameters in Eq 10 can exceed unity.

A set of design charts was presented for rectangular and trapezoidal channels with a side slope of 0.0, 0.5, and 1.0. Solutions obtained from design charts derived in this study are identical to the Hinds' graphical solutions. However, it significantly improves the accuracy and efficiency of the graphical method. This study concludes that when F_q/S_o is between 0.26 to 17.45 and N/S_o is between 0.01 and 0.50, a trapezoidal channel with a side slope of 1V:1H shall be selected. On the contrary, when F_q/S_o is between 0.0950 and 0.83 and N/S_o is between 0.005 and 0.05, a rectangular channel shall be considered.

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