

## RISK-COST APPROACH TO SIZE DETOUR CULVERT

James C. Y. Guo

Professor, Department of Civil Engineering, University of Colorado at Denver  
Campus Box 113, 1200 Larimer Street, Denver, CO 80217-3364

**Abstract:** An interim drainage structure provides a short service during the construction months of the permanent drainage structure. This study presents a risk cost approach by which the design flood for an interim drainage structure can be derived from the capacity of the permanent structure, cost-capacity ratio, cost-damage ratio, and construction months during a year. To accommodate various construction periods in a year, the design capacity for such a temporary drainage structure can be adjusted according to the monthly rainfall or runoff distribution at the project site. Design charts were produced for the break-even economic condition using the Gumbel distribution for annual maximum analysis. Similar solutions were also achieved using the Exponential distribution for annual exceedance analysis. Design methodology presented in this paper provides an objective basis for selecting construction months and sizing the interim drainage structures.

### INTRODUCTION

According to the cost data published by the Colorado Department of Highways (Cost Estimates 1986-1992) approximately one dollar out of four was spent for drainage structures in highway constructions. This is also true at a national level. During the construction of a permanent highway drainage structure, it often requires a temporary culvert to maintain the continuity of traffic flow and runoff flow. In general, an interim drain only serves through a construction period less than a year in most cases. Design of such an interim drainage structure must begin with the decision on the selection of design flood. The temporary nature in an interim phase and the potential flooding risk make this task difficult. Due to its short service, the cost of an interim drainage facility should be kept as economical as possible. However, the failure of an undersized interim drainage structure may cause as much damage, in terms of traffic loss, as losing the permanent structure.

Drainage design criteria set forth for the selection of design flood frequency are based on the importance and costs of the highway under design. For instance, the roadway design manual published by the Colorado Department of Transportation recommends that a 100-year flood be used to size drainage crossings under an urban freeway while a 25-year flood is used for sizing drainage crossings under a two-lane rural highway. These design criteria are developed for permanent structures, but not suitable for temporary drainage facilities because of the risk and cost considerations. From the risk point of view, "the larger, the better." , whereas from the cost point of view, "the smaller, the better." Preferable to rigid design guidelines is an approach in selecting a design flood frequency that achieves an optimal design using risk-cost analysis. To avoid the difficulty in cost and damage assessment in dollars, this study presents a risk-cost approach using the cost-damage ratio, cost-capacity ratio, and relative occurrence probability as decision variables. Although the method was calibrated with cost data from the State of Colorado, these nondimensional ratios may be applied in other regions.

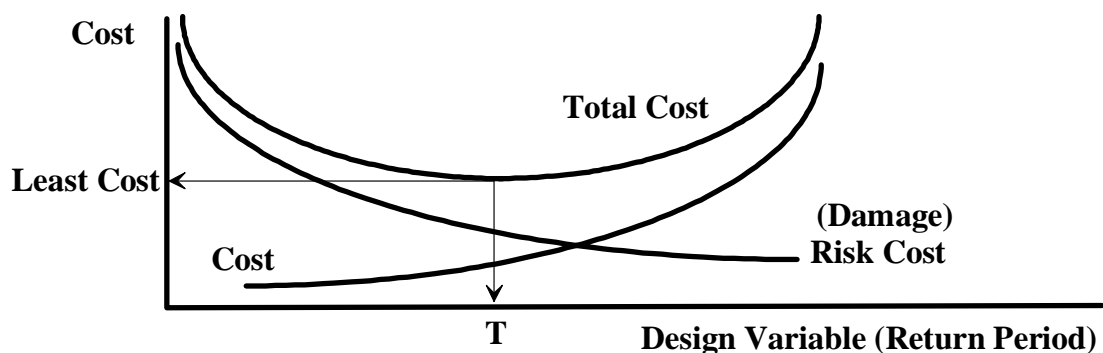


Figure 1. Optimization of Total Cost for Drainage Structure

## DEVELOPMENT OF RISK-COST METHODOLOGY

Based on the fact that the cost of a drainage structure increases with respect to its capacity, the cost of installing a drainage culvert,  $C$ , can be related to its capacity,  $Q$  (Collier et al. 1988) as:

$$C = F(Q) \quad (1)$$

in which  $F(Q)$  = a functional relationship between the cost and capacity of a highway drainage structure. Cost data published by the Colorado Department of Transportation (CDOT) from 1986 to 1992 indicated that many major drainage structures were constructed with reinforced concrete box culverts (CBC) and their interim drains were built with used corrugated metal pipes (CMP) to pass the 2- or 5-year event, depending on the on-site judgment. During an interim phase, a temporary culvert will only serve for several months. Guo (1985) concluded that the capacity of a temporary culvert is expected not to exceed a ten-year flood in most cases. Within such a narrow solution domain, the cost ratio between two culverts may be related to their capacity ratio. In this study, a linear relationship is adopted as

$$\frac{C_d}{C_p} = \alpha \frac{q}{Q} \quad (2)$$

in which  $C_d$  = cost of the interim culvert,  $C_p$  = cost of the permanent drainage structure,  $q$  = interim culvert capacity,  $Q$  = capacity of the permanent drainage structure,  $\alpha$  = the cost-capacity coefficient. Analysis of cost data published by CDOT indicated that the cost-capacity coefficient varies between 0.35 and 0.57 (Guo, 1985 and 1987). Of course, the cost ratio in Eq 2 can also be described by other nonlinear forms as long as they fit local cost data.

One of the primary drawbacks to many risk-cost procedures is the degree of difficulty in capitalizing the monetary damage resulting from a failure. The seriousness of any traffic delay is proportional to the highway site, traffic volume, availability of alternate routes, and the overall importance of the route. As far as the losses due to the discontinuity of traffic are concerned, we may conservatively consider that the failure of an interim drainage structure may result in the same amount of damage as that incurred in the failure of a permanent structure. Consequently the total loss of losing an interim drainage structure versus that of a permanent structure differs only in the cost of the structures themselves. The chance of failure of an interim culvert can be assessed by the joint probability that includes: (1) the exceedance probability,  $P_T$ , of having a flood exceeding the capacity of the interim culvert, and (2) the occurrence probability,  $P_m$ , of such a flood to occur during the service period of the interim drain.

Assuming that the interim culvert will fail when a flood exceeds the design capacity and the two events mentioned above are independent, the expected damage due to traffic discontinuity associated with the failure of an interim culvert can be written as:

$$C_r = P_T P_m D_p \quad (3)$$

in which  $C_r$  = expected damage due to the failure of the interim culvert,  $D_p$  = losses due to the discontinuity of traffic. When the interim culvert is designed to pass a flood with a frequency of  $T$ -year, then the exceedance probability,  $P_T$ , is:

$$P_T = \frac{1}{T} \quad (4)$$

The occurrence probability,  $P_m$ , of having such a flood during the construction months can be approximated by either the monthly rainfall distribution or the monthly runoff distribution normalized by its annual amount. This normalization process produces an approximation of monthly occurrence probability curve whose total area is unity. For instance, when the interim culvert is to serve from the  $i$ -th month to  $j$ -th month in a year, the occurrence probability,  $P_m$ , can be estimated as:

$$P_m = \sum_{k=i}^{k=j} \frac{\Delta P_k}{P_o} \quad \text{for } i = 1 \text{ to } 12, j = i+m, \text{ and } m < 12 \quad (5)$$

in which  $\Delta P_k$  = the k-th monthly average rainfall or runoff amount,  $i$  = the beginning month of the construction,  $j$  = the end month of the construction,  $m$  = the construction span in months, and  $P_o$  = the annual rainfall or runoff amount. Dealing with a large or rural watershed, caution shall be taken when a monthly rainfall distribution is employed because of the pronounced hydrologic delay between rainfall and runoff.

By definition, the total risk cost of an interim culvert can then be written as (Corry et. al. 1981):

$$C_T = C_d + C_r \quad (6)$$

in which  $C_T$  = total risk cost for an interim culvert. Substituting Eq's 3, 4, and 5 into Eq. 6 yields:

$$C_T = a \frac{q}{Q} C_p + P_T P_m D_P = a \frac{q}{Q} C_p + \frac{P_m D_P}{T} \quad (7)$$

As illustrated in Figure 1, the least total cost in Eq 7 can be achieved in terms of the selection of return period,  $T$ , which is the design flood frequency for the interim culvert. Mathematically, the first derivative of Eq 7 for the least cost must be equal to zero. Therefore, we have

$$\frac{dq}{dT} = \frac{Q P_m D_P}{a T^2 C_p} \quad (8)$$

Eq. 8 implies that the solution can be achieved by identifying the optimal slope on the flood peak discharge-frequency curve. In other words, to reduce the risk of failure encourages the engineer to use a larger culvert, however, to install a culvert greater than the optimal size will experience a diminishing return (Guo and Urbonas, 1996), (Urbonas et. al., 1996). According to the flood frequency analysis, the variable,  $q$ , with a return period,  $T$ , can be statistically related to its mean and standard deviation (Chow et. al. 1988):

$$q = Q_m + K_T S \quad (9)$$

in which  $Q_m$  = mean flood discharge,  $S$  = standard deviation of the flood variable, and  $K_T$  = frequency factor of the flood variable. Values of  $Q_m$  and  $S$  can be determined by the frequency analysis when runoff data are available near or at the project site. If the field data are not available, flood prediction methods may be used to estimate flood magnitudes. With any two known flood magnitudes, Eq. 9 can be simultaneously solved for mean and standard deviation. Taking the first derivative of Eq. 9 with respect to the variable,  $T$ , yields:

$$\frac{dq}{dT} = S \frac{dK_T}{dT} \quad (10)$$

Substituting Eq. 10 into Eq. 8 yields:

$$\frac{dK_T}{dT} = \frac{P_m Q D_P}{a S T^2 C_p} \quad (11)$$

Eq 11 applies to any probability distribution as long as it fits the runoff data. Runoff data used in the hydrologic peak flow frequency analysis can be structured as either annual maximum series, or annual exceedance series. Application of Eq 11 to these two types of data series is discussed as follows:

#### (a) Annual Maximum Series

The Gumbel distribution may be considered. Its frequency factor is defined as (Chow et al. 1988):

$$K_T = \frac{\sqrt{6}}{\pi} \{0.5772 + \ln \ln \left[ \frac{T}{T-1} \right] \} \quad (12)$$

in which  $\pi$  = a constant equal to 3.1416. and  $\ln$  = the natural logarithmic function. Taking the first derivative of Eq. 12 with respect to  $T$  yields:

$$\frac{dK_T}{dT} = \frac{\sqrt{6}}{\pi} \left\{ \frac{1}{\ln \left[ \frac{T}{T-1} \right] (T-1) T} \right\} \quad (13)$$

Substituting Eq. 13 into Eq. 11 yields:

$$P_m = B \left\{ \frac{T}{T-1} \right\} \frac{C_p}{D_p} \quad (14)$$

$$B = \frac{a\sqrt{6}}{\pi} \left[ \frac{S}{Q} \right] \quad (15)$$

(b) Annual Exceedance Series

The Exponential distribution is considered. Its frequency factor is defined as (Chow et al. 1988):

$$K_T = \frac{\sqrt{6}}{\pi} (\ln T - 0.5772) \quad (16)$$

Taking the first derivative of Eq 16 with respect to T yields:

$$\frac{dK_T}{dT} = \frac{\sqrt{6}}{\pi} \frac{1}{T} \quad (17)$$

Substituting Eq 17 into Eq 11 yields a linear relation as:

$$P_m = BT \frac{C_p}{D_p} \quad (18)$$

The cost to damage ratio in Eq's 14 and 18 can be determined by economical and social considerations. For instance, if the failure of the interim culvert is not tolerable for the site, a conservative design can be achieved by using a smaller ratio of  $C_p/D_p$ . On the other hand, a higher damage ratio is suggested to be unity because it is the break-even point. Although the cost to damage ratio in Eq's 14 and 18 relies on the engineer's judgment, it provides an objective basis for comparisons among alternatives. For instance, when the construction is to be in a rainy season, Eq's 14 and 18 will suggest a higher T because of a higher occurrence probability,  $P_m$ . Eq 15 also indicates that the value of B is inversely proportional to the design discharge of the permanent structure. As a result, Eq's 14 and 18 recommend a larger interim culvert for a larger permanent bridge because of the higher potential damage associated with losing the structure. In practice, the commencement of a construction phase is often not assured during the bidding process, therefore a total of 12 possible construction periods in a year need to be developed for comparison. Figures 2 and 3 are produced using Eq 14 for obtaining the design flood frequency, T, when B,  $P_m$ , and  $C_p/D_p$  are specified.

## DESIGN SCHEMATICS FOR A BRIDGE REPLACEMENT PROJECT

The existing bridge, Number N-10-C, is located at State Highway 160 and the South Fork River near Creed, Colorado. The bridge is 165 feet long and has a skew angle of 32 degree to the centerline of the river. This bridge was built in the early 1930's and is to be replaced with concrete culverts. It will take three months to complete the construction. As a four-lane highway in a rural area, the replacement bridge is designed to survive a 50-year flood. The mean and standard deviation of the annual maximum peak runoff series collected near the site were derived to be 1516.6 cfs and 754.4 cfs respectively. The magnitude of a 50-year flood is then determined to be 3472 cfs.

The USGS Water Resources Data for Colorado provide monthly runoff records from 1961 through 1997 near the project site. The monthly average runoff rates are listed as follows:

Month	Jan	Feb	Mach	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Runoff in cfs	56	62	453	574	1,550	1,470	770	375	284	198	89	75
$\Delta P_k/P_o$	0.010	0.011	0.027	0.101	0.274	0.260	0.136	0.066	0.050	0.035	0.016	0.013

**Table 1. Flood Occurrence Probability Approximated by Monthly Runoff Distribution**

The sum of these monthly runoff rates,  $P_o$ , is 5656 cfs which is then used to normalize the monthly runoff rates to obtain the approximate flood occurrence probability distribution. Applying Table 1 to the construction period from April through June as an example, the value of  $P_m$  in Eq 5, is:

$$P_m = 0.101+0.274+0.260=0.635 \quad (19)$$

This approximation can be interpreted as a chance of 63.5% to have a flood exceeding the culvert capacity within these three months in a year. Substituting a value of 0.5 for the cost-capacity ratio into Eq 15, we have  $B = 0.0847$ . Substituting the values of  $B$  and  $P_m$ , into Eq 14 results in a recommended capacity of 6.6-year flood for the interim culvert when  $C_p/D_p = 1.0$  (see Figure 2). Table 2 is the summary of design floods for 12 different construction periods in a year for  $C_p/D_p=1.0$ .

START	END MONTH OF CONSTRUCTION											
	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
JAN	1.1	1.1	1.1									
FEB		1.1	1.1	1.1								
MAR			4.5	4.5	4.5							
APR				6.6	6.6	6.6						
MAY					7.0	7.0	7.0					
JUN						4.6	4.6	4.6				
JUL							1.9	1.9	1.9			
AUG								1.1	1.1	1.1		
SEP									1.1	1.1	1.1	
OCT										1.1	1.1	1.1
NOV	1.1										1.1	1.1
DEC	1.1	1.1										1.1

The value of 1.1 represents that the recommended return period  $\leq$  One year.  
The construction period is three months.

**Table 2. Recommended Design Flood Frequency for Various Construction Periods.**

However, considering that this project site is in a rural area, the failure of an interim culvert only results in a replacement cost for the interim culvert, i.e.  $D_p=C_d$ . Under this assertion, aided by Eq 2 with  $\alpha=0.5$ , the cost-damage ratio is estimated as:

$$\frac{C_p}{D_p} = \frac{C_p}{C_d} = \frac{Q}{\alpha q} = 2.0 \frac{Q}{q} \leq 2.0 \quad (20)$$

It is expected that the ratio of  $Q/q$  is greater than unity. To be conservative, the ratio of  $C_p/D_p = 2.0$  can be used to develop alternatives. As shown in Figure 3, the design flood for the same construction period as that from April to June is reduced to a 2.9-year flood when  $C_p/D_p=2.0$ .

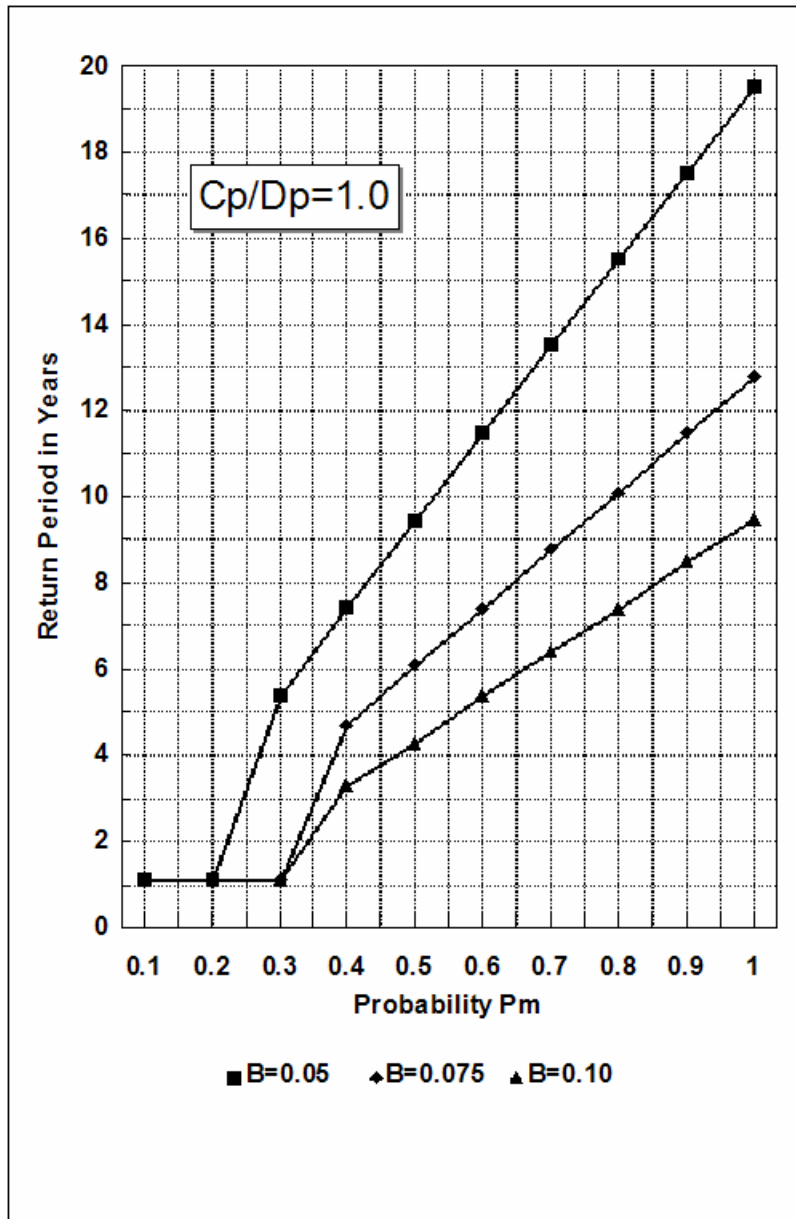


Figure 2. Design Flood Frequency For Sizing an Interim Culvert with  $C_p/D_p=1.0$  Using Gumbel Distribution

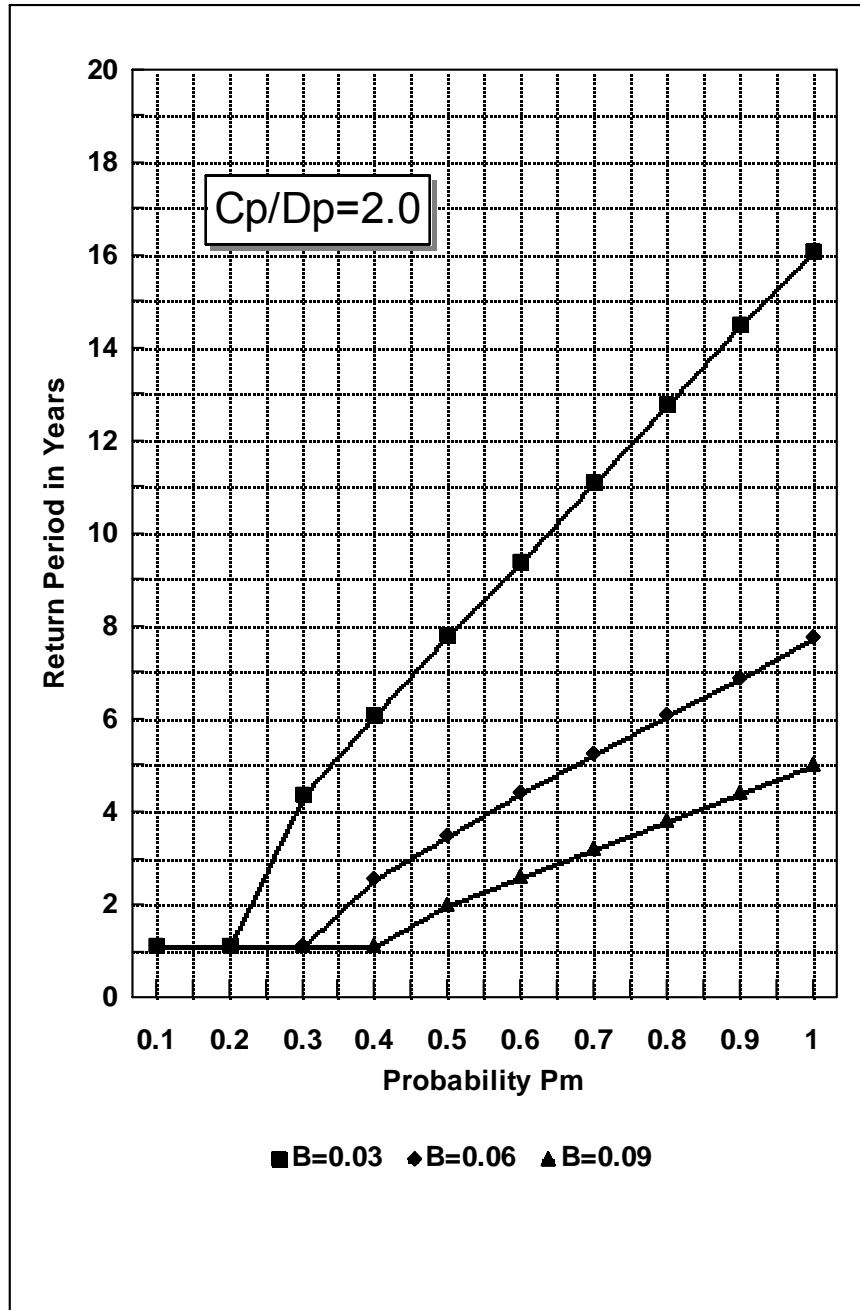


Figure 3. Design Flood Frequency For Sizing an Interim Culvert with  $C_p/D_p=2.0$  Using Gumbel Distribution

**CONCLUSIONS**

A risk-cost method was developed to size interim drainage structures by minimizing the total risk cost, including construction cost and expected failure damage. The size of an interim drainage structure can be related to the capacity of the permanent drainage structure, construction months in a year, culvert cost-capacity ratio, and cost to damage ratio. This design procedure is computerized into the model of Interim Culvert Risk Analysis, i.e. ICRAM, for design convenience. Both cost-damage ratio and cost-capacity ratios are two dimensionless parameters, and form the decision making framework. Although the economic break-even concept suggests that the cost-damage ratio be unity, ICRAM allows engineers to apply different cost-damage ratios in response to the social and economic considerations.

The optimization procedure derived in this study is inclined to recommend a larger interim culvert for a larger permanent structure, or a lower cost-capacity ratio.

The design frequency for an interim culvert using the Gumbel distribution can be obtained through an iterative process, whereas the solutions using the Exponential distribution are explicit and linear. Design charts are developed for the  $C_p/D_p$  ratio of one and two using the Gumbel distribution. However, Eq 11 is applicable to any probability distributions as long as they fit the local runoff data. In case of lacking the analytical solutions for the selected probability distribution, a numerical finite difference approach can be used to identify the gradients as described in Eq 11. As demonstrated in the design example, ICRAM can develop up to 12 design alternatives for the project, one for each construction period in a year. This approach provides an objective basis for selecting design storm event and construction period as well.

## **REFERENCES**

1. American Water Resources Council, Guideline for Determining Flood Flow Frequency, Bulletin 17B, Washington D.C., 1976, 1977.
2. Chow, Ven T., Maidment, David R., Mays, Larry W., Applied Hydrology, Chapter 12, McGraw Hill, New York, New York, 1988
3. Collier, Courtland A., and Ledbetter, William B., Engineering Economic and Cost Analysis, Chapter 12, Harper and Row Publishers, New York, New York, 1988
4. Colorado Department of Highways, Colorado Standard Plans: Division of Highways M&S Standards, Denver, Colorado.
5. Colorado Department of Highways, Roadway Design Manual, Denver, Colorado.
6. Colorado Department of Highways, Standard Specifications for Road and Bridge Construction, Denver, Colorado.
7. Corry, M.L., Jones, J.S., and Thompson, P.L., The Design of Encroachment on Flood Plains Using Risk Analysis, FHA, HEC 17, Washington, D.C., 1981.
8. Cost Estimates Squad of the Staff Design Branch, Colorado Department of Highways, 1986-1992 Cost Data, Denver, Colorado.
9. Guo, C.Y., Detour Culvert Sizing, Proceedings of Fourth International Hydrology Symposium, Fort Collins, Colorado, August, 1985.
10. Guo, C.Y., Development of Risk-Cost Methodology for Detour Culvert Design, the Research Contract Number 1503A, CDOH-UCD-R-88-5, Colorado Department of Highway, Denver, Colorado, December 1987.
11. Guo, C.Y., Urbanas, Ben R., Maximum Detention Volume Determined by Runoff Capture Ratio, ASCE journal of Water Resources Planning and Management, Vol 122, No 1, ISSN 0733-9496, Jan/Feb 1996, pp33-pp39.
12. Guo, C.Y. "Technical Manual for Highway Interim Culvert Risk Analysis Model", Department of Civil Engineering, University of Colorado at Denver, Denver, Colorado, August, 1996.
13. Ben R. Urbanas, Rosener, Larry A., Guo, C.Y., Hydrology for optimal Sizing of Urban Runoff Treatment Control Systems, Water Quality International, Jan/Feb 1996, pp30-pp33.
14. US Geological Survey, Water Resources Data Colorado, Water Year 1961 to 1977.