Guo, James C.Y. (1998). "*Overland Flow on a Pervious Surface,*" IWRA International J. of Water, Vol 23, No 2, June.

Kinematic Wave Solution for Overland Flow On Pervious Surface

by
James C.Y. Guo, PhD, P.E.
Professor, Civil Engineering, U. of Colorado at Denver, Denver, Colorado 80217, James.Guo@cudenver.edu

Abstract: This paper presents the analytical solutions of overland flow on a pervious surface under a uniform rainfall with decay infiltration. It was found that the kinematic wave travel time through a small catchment under such a non-uniform rainfall excess is not a constant, but varies between the time of concentration and the time of equilibrium because of the soil antecedent moisture conditions. Derivations of the analytical solutions reveal that kinematic wave travel times are part of the hydrograph convolution process, and can not be directly measured. As a result, this paper challenges the reliability of those empirical time of concentration formulas developed for and calibrated by the time difference between the center of mass of the rainfall excess and the inflection point on recession of the observed runoff hydrograph.

INTRODUCTION

The time of concentration of a small catchment is defined as the period of time required for water to travel from the most upstream boundary to the catchment outlet. It is an important parameter for modeling the rainfall and runoff process in a small watershed because it presents a reference to judge if the rainfall duration is long enough for the entire watershed to contribute to runoff. McCuen et al (1984) evaluated several empirical equations developed for estimating the time of concentration. Most of them consider that the time of concentration is a constant period of time, and is only related to the catchment drainage characteristics such as watershed slope, flow length, and surface roughness (Katz et al 1995). In fact, kinematic wave movements in a catchment also depend on rainfall amount, runoff depth and some other minor factors.

Wooding (1965) applied the kinematic wave approach to a rectangular catchment and derived the time of equilibrium of the catchment under temporally and spatially uniform rainfall excess. Wooding's solutions do not differentiate the time of equilibrium from the time of concentration (Chow 1964). However, with a non-uniform rainfall distribution, the greater rainfall excess results in faster flood flow and a shorter flow time. In this study, this fact is demonstrates by applying the kinematic wave approach to overland flow on a pervious area. Using the empirical Horton decay formula for soil infiltration losses, kinematic wave solutions indicate that at the beginning of a rainfall event, higher infiltration rates result in less runoff rates and longer travel times. Tne travel time through the catchment is eventually reduced to the time of equilibrium when the infiltration rate reaches its constant value. Under a nonuniform rainfall, the time of concentration is part of hydrograph convolution process, and is not so simple as the time difference between the center of mass of rainfall excess and the inflection point of recession of the direct runoff hydrograph.

OVERLAND FLOW ON A PERVIOUS SURFACE

The one-dimensional dynamic wave equations for overland flow under a uniform rainfall and decay infiltration losses are (Yen and Chow 1974):

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial y}{\partial x} = g(S_o - S_f) \tag{1}$$

$$\frac{\partial y}{\partial t} + \frac{\partial q}{\partial x} = i_e(t) \tag{2}$$

$$i_e(t) = i - f(t) \tag{3}$$

$$f(t) = f_c + (f_o - f_c)e^{-kt} \tag{4}$$

in which t = time, x = distance along the flow path, u = flow velocity, g = gravitational acceleration, S_o = overland slope, S_f = friction slope of the flow, q = runoff rate per unit width, y = runoff depth, i = uniform rainfall intensity, $i_e(t)$ = rainfall intensity excess, f (t) = infiltration rate at time t, fo = initial infiltration rate, fc = final infiltration rate, and k = soil infiltration decay coefficient.

Assuming that the gravitational force in an overland flow is balanced by the friction force, Eq 1 is simplified to the kinematic wave equation of motion:

$$S_O = S_f \tag{5}$$

Eq 5 implies a relationship similar to Manning's equation between flow rate and flow depth, namely:

$$q = \alpha y^m \tag{6}$$

in which α = conveyance factor that is determined by the overland slope, S_o and surface Manning's roughness n, and m = empirical exponent that varies between 3/2 to 3, depending on the flow regime, turbulent or laminar. Buetner et al. (1940) reported that in their 45 laboratory tests, the average value of m was found to be 1.85. For a wide overland flow, Woolhiser and Liggett (1967) , Eagleson (1970), and Buetner et al. (1940) recommended a value of 2 for m. As a result, Eq. 6 becomes

$$q = \alpha y^2 \tag{7}$$

This study accepts the above recommendation for further investigating the role of the time of concentration in kinematic wave movements. Taking the partial derivation of Eq 7 with respect to x yields

$$\frac{\partial q}{\partial x} = 2\alpha y \frac{\partial y}{\partial x} \tag{8}$$

Substituting Eq 8 into Eq 1 yields

$$\frac{\partial y}{\partial t} + 2\alpha y \frac{\partial y}{\partial x} = i_e(t) \tag{9}$$

Eq 9 is the total derivative for the flow depth. Therefore, we have

$$\frac{dy}{dt} = i_e(t) \tag{10}$$

As shown in Eq 9, the celerity of the kinematic wave in Eq 10 is

$$\frac{dx}{dt} = 2\alpha y \tag{11}$$

The initial condition and boundary condition for Eq's 10 and 11 are

$$y = 0 \quad for \quad 0 \le x \le L \quad for \quad t = t_s \tag{12}$$

$$y = 0 \quad for \quad x = 0 \quad for \quad t \ge 0 \tag{13}$$

in which ts = ponding time or runoff incipient time in case that fo > i (Bedient and Huber 1992), and L = length of the overland flow.

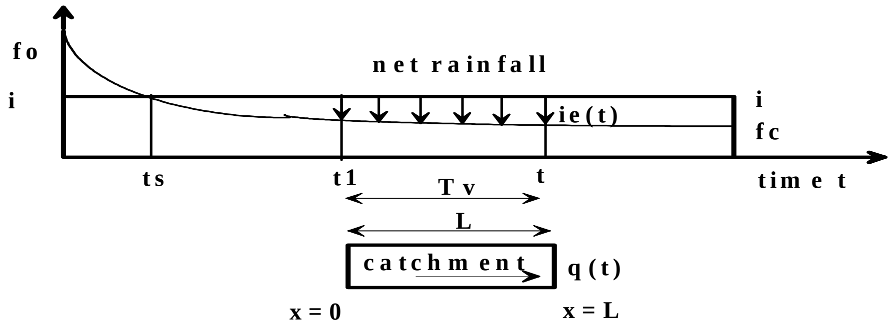

Figure 1. Illustration of integration domains for time and space.

At a given time, t, and t > ts, let Tv be the travel time for kinematic wave to move through the catchment. As shown in Figure 1, the runoff, q(t), or its depth y, at time, t, depends on how much rainfall excess received by the catchment, i.e. from x= 0 to x= L, during the travel time, Tv, i.e. from time, t-Tv, to t. For convenience, let the time variable, t_1,, be the difference, t-Tv. As a result, Eq's 10 and 11 may be integrated as:

$$\int_0^y dy = \int_{t_1}^t (i - f(t))\, dt \qquad \text{for } t_1 > ts \qquad (14)$$

$$\int_0^x dx = \int_{t_1}^t 2ay\, dt \qquad \text{for } t_1 > ts \qquad (15)$$

With the aide of Eq 4, Eq 14 can be integrated as:

$$y = (i - f_c)(t - t_1) + \frac{f_o - f_c}{k}(e^{-kt} - e^{-kt_1}) \qquad 16)$$

Substituting Eq 16 into Eq 15 yields

$$\frac{x}{2a} = \frac{(i - f_c)}{2}(t - t_1)^2 + \frac{(f_o - f_c)}{k^2}\{[1 - k(t - t_1)]e^{-kt_1} - e^{-kt}\} \qquad 17)$$

Using the definition of the time of concentration, we have

$$t_1 = t_s, \quad x = L, \quad and \quad t = T_c + t_s \qquad (18)$$

Substituting Eq 18 into Eq's 16 and 17 yields

$$Y_c = (i - f_c)T_c + \frac{f_o - f_c}{k}e^{-kt_s}(e^{-kT_c} - 1) \qquad (19)$$

$$\frac{L}{2a} = \frac{(i - f_c)}{2}T_c^2 + \frac{f_o - f_c}{k^2}e^{-kt_s}(1 - kT_c - e^{-kT_c}) \qquad 20)$$

in which Y_c = outlet depth at time, (ts+Tc). After the time period, ts+Tc, the entire catchment area becomes the tributary to the runoff at the outlet. Due to the decay of infiltration, the kinematic waves will move faster with more rainfall excess and shorter travel times. Referring to Figure 1, it is concluded

$$t_1 = t - T_v, \quad and \quad x = L \qquad for \quad t \ge (t_s + T_c) \qquad (21)$$

Substituting Eq 21 into Eq's 16 and 17 yields

$$Y_p = (i - f_c)T_v + \frac{f_o - f_c}{k}e^{-k(t - T_v)}(e^{-kT_v} - 1) \qquad (22)$$

$$\frac{L}{2\alpha} = \frac{(i-f_c)}{2}T_v^2 + \frac{f_o-f_c}{k^2}e^{-k(t-T_v)}(1-kT_v-e^{-kT_v})$$ 23)

in which Y_p = outlet runoff depth after the time of concentration. When time t is long enough for the infiltration rate to reach its final rate, Eq's 22 and 23 are reduced to

$$Y_e = (i-f_c)T_e$$ (24)

$$\frac{L}{2\alpha} = \frac{(i-f_c)}{2}T_e^2$$ 25)

in which Y_e = equilibrium runoff depth at the outlet, and T_e = the time of equilibrium. It is noted that Eq's 24 and 25 agree with Wooding's solution (17) for the overland kinematic wave flow under a uniform rainfall excess. Normalizing Eq 23 by T_e in Eq 25 yields

$$\frac{T_v^2}{T_e^2} = 1 - \frac{1}{2}\frac{(f_o-f_c)}{(i-f_c)}\frac{e^{-k(t-T_v)}}{k^2T_e^2}(1-kT_v-e^{-kT_v})$$ (26)

Figure 2 shows the impact of the soil antecedent moisture condition on the variation of the kinematic wave travel time as expressed in Eq 26. The catchment used in the study has a flow length of 400 ft, α of 5.0, rainfall intensity of 6.0 inch/hr, infiltration decay coefficient of 0.0018 per second, and final infiltration rate of 0.5 inch/hr. The incline line, AB, in Figure 2 represents the times of concentration of the catchment under various soil antecedent conditions. As expected, the drier the soil is, the longer the time of concentration will be. The line, AC, represents the time of equilibrium of the catchment. The kinematic wave travel times in the catchment varies between the time of concentration and the time of equilibrium.

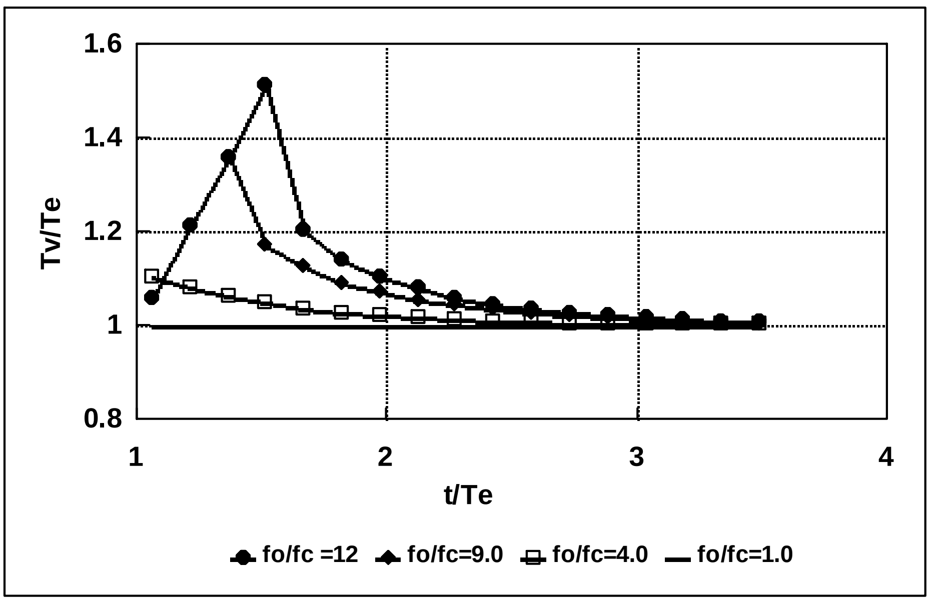

Figure 2. Variations of Kinematic Wave Travel Time for Different Soil AMC
L= 400-ft, α= 5.0, i = 6.0 inch/hr, k = 0.0018/sec, and fc = 0.5 inch/hr

OVERLAND RUNOFF HYDROGRAPH

The overland runoff hydrograph generated from a catchment under a long rainfall can be divided into three distinct portions: Rising, Peaking, and Recession. Applications of the solutions derived in this study to each portion are discussed as follows:

A. Rising Portion

As shown in Figure 3, the rising portion on the hydrograph starts from the ponding time, ts, and ends at the time of concentration, (ts+Tc). During this period of time, the rising runoff rates can be predicted by letting the time variable, t_1, in Figure 1 be the ponding time, ts. Thus, Eq's 16 and 17 become

$$y=(i-f_c)(t-t_s)+\frac{f_o-f_c}{k}(e^{-kt}-e^{-kt_s}) \qquad for \quad t_s\le t\le(t_s+T_c) \tag{27}$$

$$\frac{x}{2a}=\frac{(i-f_c)}{2}(t-t_s)^2+\frac{(f_o-f_c)}{k^2}\{[1-k(t-t_s)]e^{-kt_s}-e^{-kt}\} \quad for \quad 0\le x\le L \qquad 28)$$

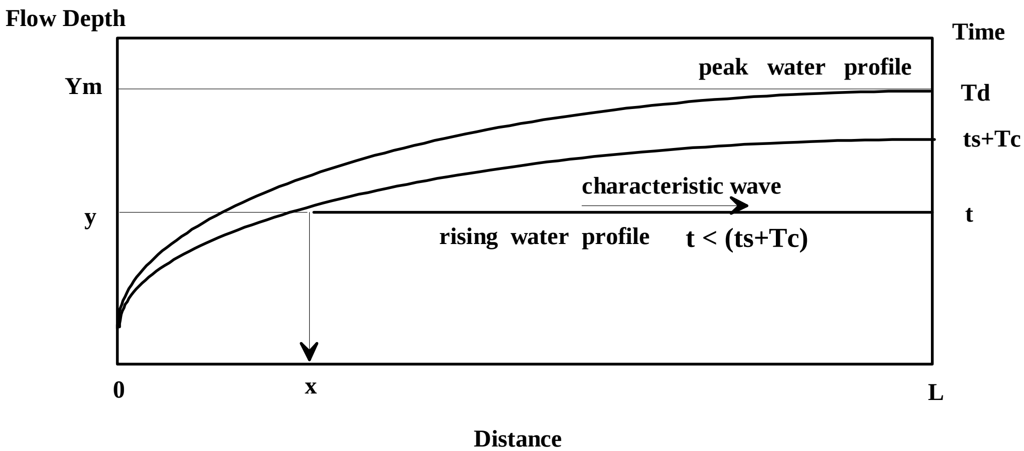

Figure 3. Rising Water Profile along the Flow Path and Outlet Hydrograph

<u>B. Peaking Portion</u>

Referred to Figure 3, the peaking portion starts at (ts+Tc) when the entire catchment becomes the tributary area and continues until, Td ,when the rain ceases. The runoff rate during the peaking portion is predicted by first determining the travel time by Eq. 23 and then rainfall excess amount by Eq 22. The peak runoff depth, Y_m, occurs at the time when the rain ceases. Therefore, we have

$$Y_m=(i-f_c)T_v+\frac{f_o-f_c}{k}e^{-k(T_d-T_v)}(e^{-kT_v}-1) \qquad for \quad t=T_d \tag{29}$$

It is noted that the depth, Y_m , asymptotically approaches the equilibrium depth, Y_e .

<u>C. Recession Portion</u>

During the recession, the flow depth at the outlet is predicted by integrating Eq 10 with i = 0 as

$$\int_{Y_m}^{y} dy=-\int_{T_d}^{t} f(t)dt \tag{30}$$

Substituting Eq's 29 and 4 into Eq 30 yields

$$y=Y_m-f_c(t-T_d)+\frac{f_o-f_c}{k}(e^{-kt}-e^{-kT_d}) \qquad for \quad t>T_d \tag{31}$$

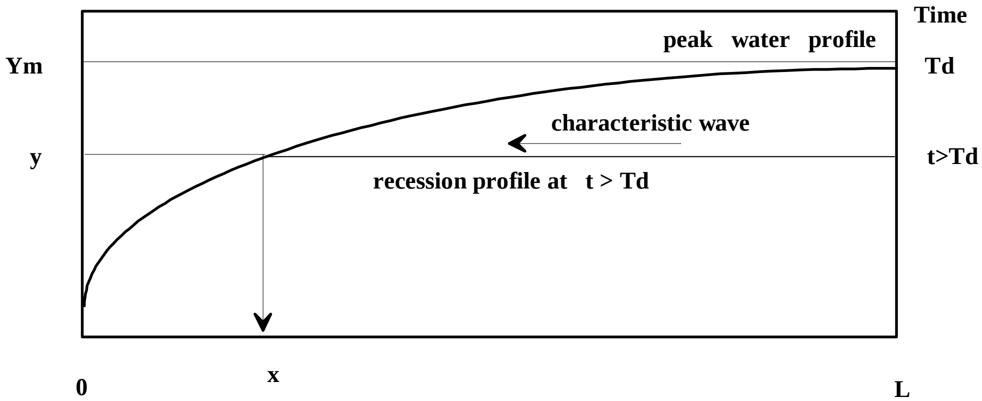

Figure 4. Recession of Water Profile along the Flow Path after Rain Ceases

In the recession, the kinematic wave recedes with a negative velocity from L to x. Therefore, the travel distance for kinematic wave is depicted as:

$$\int_{L}^{x} dx = -\int_{T_d}^{t} 2\alpha y dt \qquad (32)$$

Substituting Eq 31 into Eq 32 yields

$$\frac{L-x}{2\alpha} = y_m(t-T_d) - \frac{f_c}{2}(t-T_d)^2 + \frac{f_o - f_c}{k^2}\{[1+k(t-T_d)]e^{-kT_d} - e^{-kt}\} \quad for \quad t > T_d \qquad (33)$$

Runoff hydrographs presented in Figure 5 were produced using Eq's 27 through 29 for various soil antecedent moisture conditions on the same catchment used in Figure 2. The special case in which $f_o = f_c$ repeats the Wooding's solutions (1965), (1967). It is noticed that the steep increase on the rising hydrograph before (ts+Tc) was caused by the incremental contributing area to the runoff as depicted in Eq's 27 and 28. After (ts+Tc), the mild increase on the peaking hydrograph was caused by the reduction of infiltration rates. Figure 6 presents a comparison between laboratory measurements (Beuter et al in 1940, Holden et al in 1995) and predictions from Eq's 27 through 33. Good agreement is achieved.

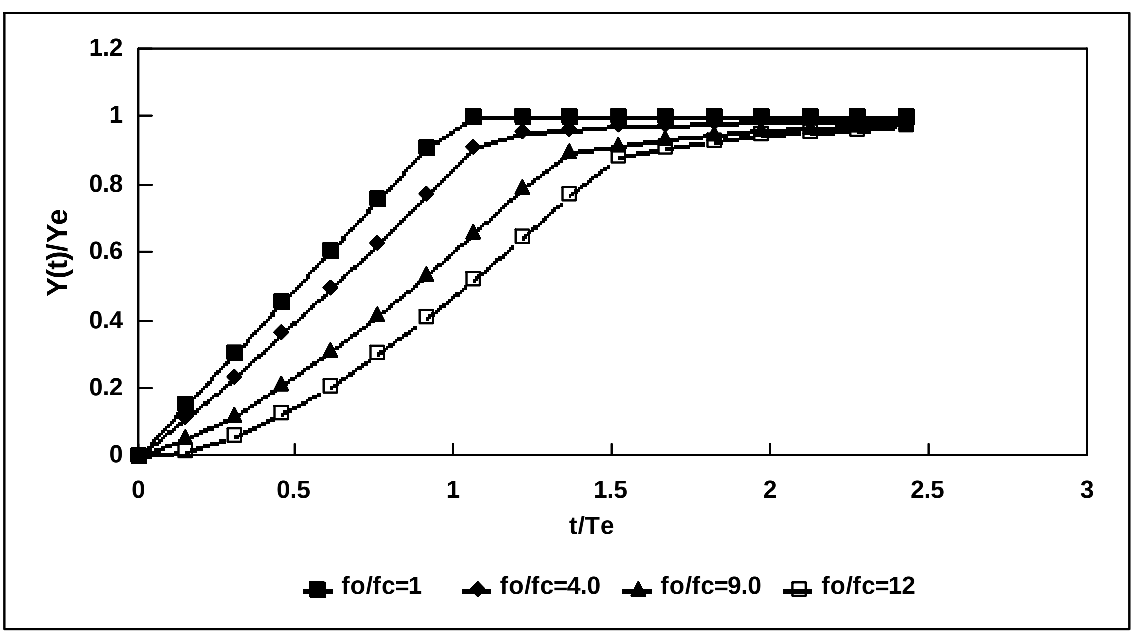

Figure 5. Overland Flow Hydrographs Under Various Soil Antecedent Conditions
L= 400-ft, α= 5.0, i = 6.0 inch/hr, k = 0.0018/sec, and fc = 0.5 inch/hr

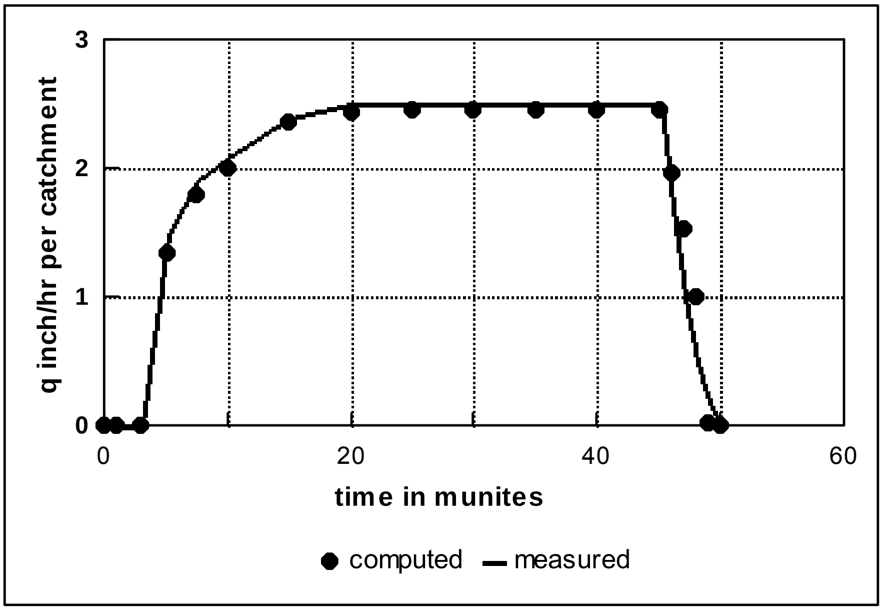

Figure 6. Comparison between Predicted and Measured Hydrographs
i=3.36 inch/hr, L=15.6 ft, k=0.0053 per second, fc = 0.91 inch/hr, fo=5.81 inch/hr, α=6.04

CONCLUSION

The rate of storm runoff generated from a catchment can not exceed the rainfall excess received during the period of time that the flood wave travels through the catchment. In practice, the time of concentration is used to warrant the consistency between the length of the waterway in a catchment and the contributing duration on a rainfall distribution. Under a uniform rainfall, the kinematic wave travel time through a catchment is a constant. However, under a non-uniform rainfall, the travel time is no longer a constant, but varies with respect to rainfall excess. In this study, Wooding's solutions (1965, 1967) were expanded to include the consideration of decay infiltration rate. It was found that under a uniform rainfall with a decay infiltration, the kinematic wave travel times through a catchment are reduced from the time of concentration to the time of equilibrium. The peak portion on an overland hydrograph starts at the time of concentration and continues increasing until the rain ceases. With a rain long enough to reduce the infiltration rate to its constant value, the overland hydrograph will reach its equilibrium. In fact, the time of concentration is the longest kinematic wave travel time, but results in the lowest peak runoff at the beginning of the peak portion on the overland runoff hydrograph. The time of equilibrium is the shortest kinematic wave travel time, but results in the maximum runoff that the catchment can produce for the given rainfall intensity and infiltration rates.

In practice, the time of concentration of a catchment was observed and estimated by the time difference between the center of mass of rainfall excess and the inflection point of recession of the direct runoff hydrograph. This study indicates that the time of concentration is involved in the runoff convolution process, and can not be directly measured from a hydrograph. Therefore, conclusions from this study challenge the reliability of those empirical formulas developed for and calibrated by the time difference defined as the above on the observed hydrographs. Recognizing the role of the time of concentration in runoff convolution process, it is necessary to examine or even revise those empirical formulas developed for estimating the time of concentration for small catchments.

Appendix -I REFERENCES

Bedient, P.B. and Huber, W.C., "Hydrology and Floodplain Analysis", 2nd Edition, Addison Wesley Publishing Company, New York, 1992

Beuter, E.L., Gaebe, R.R. and Horton, R.E., "*Sprinkled-Plat Runoff and Infiltration-Experiments on Arizona Desert Soils*", Trans, Amer. Geophys. Union, pp 550-558, 1940.

Chow, V.T., "*Handbook of Applied Hydrology*", McGraw-Hill Book Company, Chapters 17 and 21, 1964
Eagleson, P.S., "*Dynamic Hydrology*", McGraw Hill, New York, pp 344-346, 1970

Holden,A.P. and Stephenson, D., "Finite Difference Formulations of Kinematic Equations", J. of Hydraulic Engineering, ASCE, pp 423-426, May, 1995

Katz, D. M., Watts, F. J., and Burroughs E.D., "*Effects of Surface Roughness and Rainfall Impact on Overland Flow*", J. of Hydraulic Engineering, ASCE, Vol 121, No 7, July 1995.

Lighthill, M.H. and Whitham, G.B., "*On Kinematic Waves, I, Flood Movement in Long Rivers*", Proc., Royal Soc., of London, Ser. A., Vol 229, pp 281-316, 1955

McCuen, R. H., Wong, S. L., and Rawls W. J., "*Estimating Urban Time of Concentration*", J. of Hydraulic Engineering, ASCE, Vol 110, No. 7, July 1984.

Morgali, J.R., "*Laminar and Turbulent Overland Flow Hydrographs*", J. of Hydraulic Engineering, ASCE, HY 2, pp 441-360, 1970

Morgali, J.R., and Linseley, R.K., "*Computer Analysis of Overland Flow*", J of Hydraulic Engineering, ASCE, HY 3, pp 81-100, 1965.

Urban Highway Storm Drainage Model, " *Inlet Design Program*", Vol 3, Federal Highway Administration, Report No. FHWA/RD-83/043, December 1983.

Urban Drainage and Design Criteria, Denver Urban Drainage and Flood Control District, 1967.

Wooding, R.A, "*A Hydraulic Model for a Catchment-Stream Problem*", J. of Hydrology, Vol 3, pp 254-267, 1965

Woolhiser, D.A., and Liggett, J.A., "*Unsteady One-dimensional Flow over a Plane - the Rising Hydrograph*", Water Resources, Vol 3, No 3, pp 753-771, 1967

Yen, B.C. and Chow, V.T., "*Experimental Investigation of Watershed Surface Runoff*", Hydraulic Engineering Series, No 29, Dept of Civil Engineering, U of Illinois at Urbana- Champaign, September 1974.