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Application of Potential Flow Model to Water Mound under an Infiltration Basin

James C.Y. Guo, PhD., P.E.,
Department of Civil Engineering, University of Colorado at Denver, Denver, CO 80217-3364.
E-mail: jguo@carbon.cudenver.edu

Abstract: *In this study, the potential flow model developed for the infiltrating water under a basin has been expanded to predict the growth and recession processes of a water mound. Assessment of the height of a water mound always involves the selection of the soil hydraulic conductivity which in fact varies through a wet and dry cycle. In this study, analyses of nine cases provide a data base to calibrate the potential flow model. The representative hydraulic conductivity can be correlated to its saturated hydraulic conductivity by infiltration rate, size of basin, and thickness of aquifer. The potential flow model developed in this study is suitable for small storm water retention basin designs. With a proper unsaturated soil hydraulic conductivity, the potential flow model can be a useful and practical tool for estimating water mounding impacts.*

Key Words: *circular basin, trench, infiltration, hydraulic conductivity, water mound*

INTRODUCTION

Infiltration of storm water through soils is a transient flow process. When an infiltration basin is loaded with storm water, the soil medium between the basin and the groundwater table will undergo a storage process in which the soil water moisture varies from an unsaturated to a saturated condition. When the infiltrating water reaches the groundwater table, a water mound will begin to build up. The shape and growth of a mound depend on the infiltration rate, size of the basin, and hydraulic properties of soil medium (Ferguson 1990).

The water mounding process has been investigated using high mathematical skills (Hantush in 1968, Ortiz, et al. in 1979, Morel-Seytous, et al. in 1988 and 1990). They are not easy to use and require appropriate assumptions. For instance, the linearization term in Hantush's solution needs to be approximated by the aquifer thickness which results in less predictions than those obtained by the true effective thickness (Press, et al. 1989). To improve analytical solutions, a unsaturated to saturated flow equation was formulated to solve for cross sectional flows by the finite difference numerical model, VS2D, (Lappala et al. 1987). Differences between the VS2D model and Hantush's solution for several scenarios can be as high as 25% because of the assumption to cope with the soil storage effect in the unsaturated zone (Sumner et al. 1999).

Although water mounding is a transient process, it may reach a quasi steady state when the inflow and outflow rates are balanced. For instance, the numerical simulations of the water mound below a 500 m by 100 m rectangular basin indicate that the water mound begins with a fast and significant buildup in the first 30 days, and then gradually reduces its height for the next 52.3 days until a steady state is achieved (Rastigo and Pandy 1998). Such a cycle of increase and decrease in mound height was caused by the change of water content in the soil. During the initial period, the soil is unsaturated with a lower hydraulic conductivity, and the soil water storage capacity has a significant effect on the growth of water mound. As the time of recharge increases, the growth of a mound on a groundwater table results in higher hydraulic gradients from the top of the mound toward the initial water table. The mound growth reaches its steady state when the rate of infiltration is equalized by the rate of recharge to the groundwater table (Bouwer 1999). With the concept of equilibrium in flow rates, the surface-subsurface potential flow model was developed to estimate the height of a water mound underneath an infiltration trench (Guo 1998). The required subsurface geometry of a trench was found to be related to the half width of the trench and the ratio of infiltration rate to hydraulic conductivity. Although this model provides good agreements with the predictions by the numerical model of MODFLOW (McDonald et al. 1984), the selection of a proper value for the unsaturated hydraulic conductivity remains as a judgment without any guidance.

In general, the soil hydraulic conductivity can be related to its saturated value by the soil-water characteristic curve (Stankovich and Lockingtom 1995). The modeling of infiltrating water through soils typically requires such a functional representation of soil hydraulic properties (Brook and Corey 1964). During a water mounding process, the soil medium undergoes a wetting and draining cycle. The soil hysteretic functional relationship between soil moisture content and pressure head makes the determination of hydraulic conductivity even more challenging. In previous studies, both soil characteristic curve (Sumner et al. 1999) or a constant hydraulic conductivity (Bouwer in 1999, Rastogi and Pandy in 1998) have been used to model a water mound. In this study, the concept of a constant hydraulic conductivity is adopted to expand the potential flow model to include the growth and recession processes of a water mound beneath a trench or a circular basin. Comparisons with nine published cases provide a data base to calibrate the potential flow model. The representative soil hydraulic conductivity for a water mounding process can be well correlated to the soil saturated hydraulic conductivity by infiltration rate, size of basin, and thickness of aquifer. With such a quantifiable guidance, the potential flow model provides a simplified and useful approach for basin site evaluation and water mound assessment.

POTENTIAL FLOW MODEL FOR A TRENCH

As illustrated in Figure 1, the infiltrating water begins with a vertical downward velocity through the unsaturated zone underneath the basin. As the soil water content increases, the diffusive nature of the wetting front results in flow movements in both vertical and lateral directions. Many studies on water mounds were performed with a potential function using the groundwater depth as the variable (Hantuch 1968, Bouwer 1999). On the other hand, the movement of a seepage flow through soils can also be depicted by a potential flow using a stream function. With the consideration of vertical and lateral movements, the stream function was developed for the infiltrating flow under a trench as (Guo 1998):

$$\psi = \frac{f}{D} xy \qquad (1)$$

in which ψ = stream function, f = infiltration rate, D = saturated depth, x = horizontal from the central axis of the basin, and y = vertical distance below the basin. Eq 1 satisfies the two dimensional continuity and irrotationality equations. The stream function value, ψ_c , at point C (B,D) is

$$\psi_c = q = fB \qquad (2)$$

in which q = infiltration volume rate, and B = half width of the trench. To maintain the continuity along the streamline, ψ_c , between the basin and any cross section at a distance y, Eq 1 yields

$$w = \frac{BD}{y} \qquad (3)$$

in which w = width of wetting front, and y = vertical distance to width w. Derivatives of Eq 1 with respect to y and x represent the velocity vectors in the flow field as:

$$u = \frac{f}{D} x \qquad (4)$$

$$v = -\frac{f}{D} y \qquad (5)$$

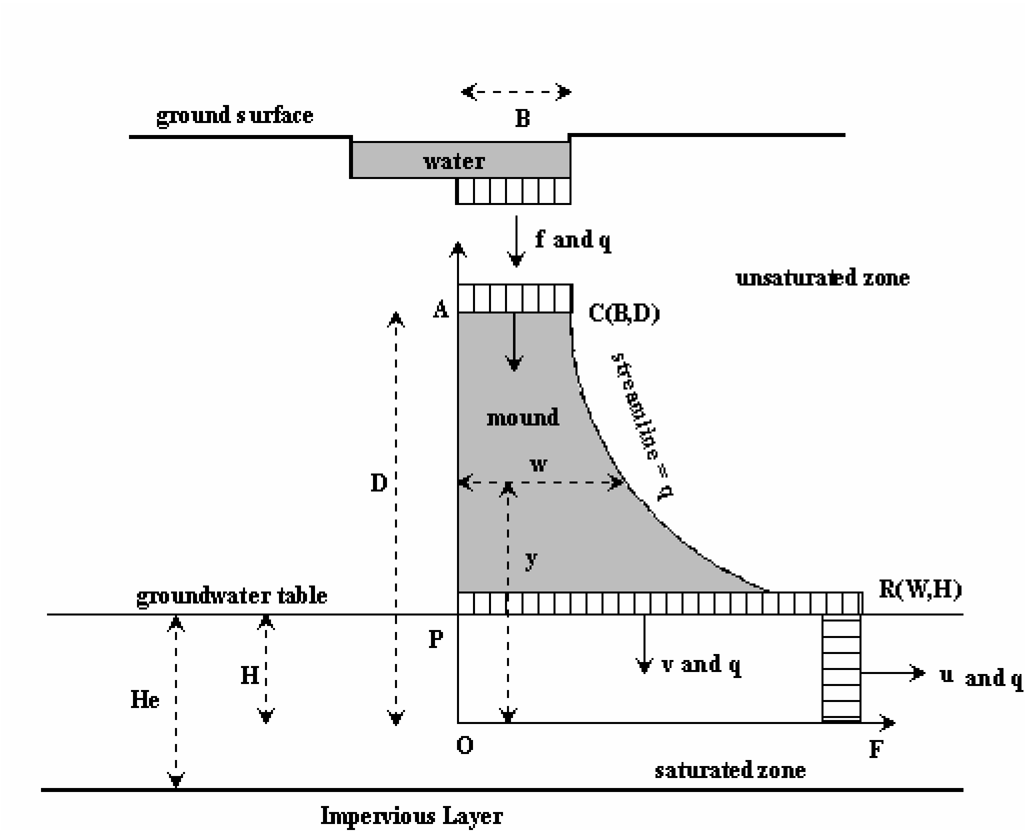

Figure 1 Illustration of Subsurface Flow Under Basin

STEADY STATE FOR INFILTRATING FLOW BELOW A TRENCH

Under the condition of a steady state, the flow rate remains as a constant at Sections AC, PR, and RF in Figure 1. On Section PR, the hydraulic gradient, i, can be approximated by unity, i = -1 (downward) when the soil suction is ignored (Bouwer 1999, Guo 1998). According to the Darcy's law, the flow of recharge at Section PR is:

$$v = \frac{q}{W} = \frac{K_y i W}{W} = -K_y \quad (6)$$

in which K_y = hydraulic conductivity in the vertical direction. Eq 6 must agree with Eq 5 at y=H. As a result, it is concluded that

$$D = \lambda_y H \quad (7)$$

$$\lambda_y = \frac{f}{K_y} \quad (8)$$

Streamlines of recharge flow near the mound are concentrated in the upper or active thickness of the aquifer, with much less flow and almost stagnant water in the deeper portion of the aquifer (Bouwer 1999). In this study, such an active flow depth, H, below the initial groundwater table is estimated by the Depuit-Forchheimer equation as:

$$q = \psi_c = \frac{K_x}{2}\frac{(D^2 - H^2)}{(W - B)} \tag{9}$$

in which K_x = hydraulic conductivity in the lateral direction. The difference, $(D^2\text{-}H^2)$, is an important parameter and can in fact be incorporated into the governing equation to estimate the height of a water mound (Griffin and Warrington, 1988). In general, storm water infiltration basins are small in size and operated with short loading periods. Unlike the long term recharging basin, the height of a water mound under a small storm water retention basin decays toward the initial groundwater table. Under this condition, Eq 9 can be an approximation near the edge of the area of recharge, i.e. Point R (W, H) in Figure 1. Eq 9 divided by the effective flow depth, H, yields a sectional velocity which must be equal to Eq 4 at x = W. As a result, the effective thickness of aquifer is derived as:

$$\frac{H}{B} = \sqrt{\frac{2\lambda_x}{\lambda_y + 1}} \tag{10}$$

$$\lambda_x = \frac{f}{K_x} \tag{11}$$

In practice, the value of H is the smaller one between Eq 10 and the available thickness, He . Aided by Eq 7, the saturation depth is

$$\frac{D}{B} = \lambda_y \sqrt{\frac{2\lambda_x}{\lambda_y + 1}} \tag{12}$$

For convenience, an anisotropic subsurface environment can be converted to an isotropic condition by an appropriate coordinate transformation, and the equivalent homogenous hydraulic conductivity, K_u , is equal to the geometric mean as (Morris and Wiggert, 1972):

$$K_u = \sqrt{K_x K_y} \tag{13}$$

Applications of the potential flow model to water mound predictions require the priori knowledge of K_u. During a water mounding process, the value of K_u varies with respect to soil moisture content. In order to develop a guidance for determining the value of K_u , the potential model is expanded in this study into the growth and recession processes of a water mound. The available water mound data can then be employed to calibrate the model in terms of the proper value of K_u.

GROWTH FUNCTION OF WATER MOUND

Before the steady state is developed, the soil storage effect as shown in Figure 2 can be depicted as a water volume balance between the inflow at Section AC and the outflow at Section PR as:

$$\frac{dV}{dt} = q - q_o \tag{14}$$

in which V = soil water storage volume, t = time, and q_o = outflow rate to recharge. Since the infiltrating flow must maintain the continuity as described by the stream function, the water flow during the growth process of a water mound shall be within the soil volume outlined by the stream lines. Aided by Eq 3, the incremental volume, dV, at a depth y can be expressed as:

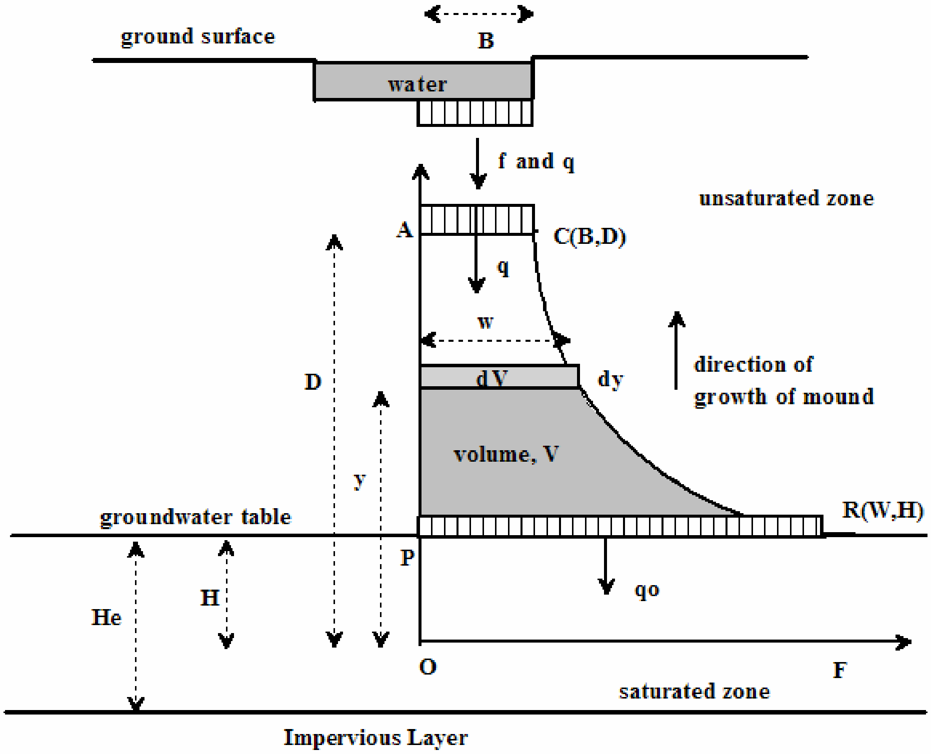

Figure 2 Illustration of Growth of Wetting Front

$$dV = S_y w dy = S_y \frac{D}{y} B dy \qquad (15)$$

in which S_y = soil yield. Similar to Eq 6, the outflow rate at Section PR is described as:

$$q_o = S_y K_u W = S_y K_u \frac{D}{H} B \qquad (16)$$

in which K_u = representative hydraulic conductivity for the growth of a water mound. Aided by Eq's 2, 15, and 16, Eq 14 can be integrated from the thickness of aquifer, H, to a depth y and yields the growth function of water mound as:

$$y = H\, e^{\frac{kt}{D}} \qquad (17)$$

$$k = \frac{f}{S_y} - K_u \frac{D}{H} \quad (18)$$

in which k = soil storage factor for the growth period of a water mound. In general, the exponent in Eq 17 is so small that Eq 17 is reduced to

$$y = H(1 + \frac{kt}{D}) \quad (19)$$

For a specified time t, Eq's 17 or 19 can be used to estimate the growth of a water mound.
Eq 17 is an increasing function with time. However the mound height, y, shall not exceed the saturated depth, D in Eq 12.

RECESSION FUNCTION OF WATER MOUND

After a water mound has been developed below the basin, during the rest period of a basin, the mound begins to recede by gradually recharging the groundwater table. As shown in Figure 3, the volume of recharge at Section PR during the recession must be equal to the volume released from the soil medium.

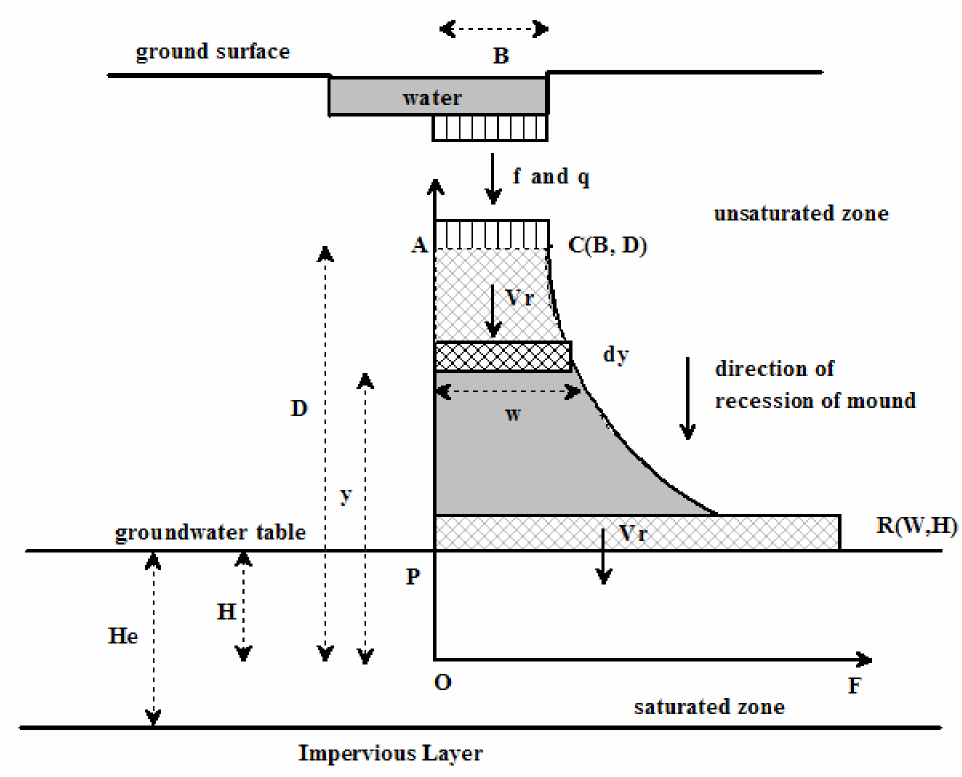

Figure 4 Recession of Water Mount

The recession of a water mound begins at the basin bottom and maintains the same shape as the saturated one. The water volume drained from the soil medium between D and y in Figure 3 is:

$$V_r = S_y \int_y^D w dy = S_y \int_y^D \frac{D}{y} B dy = S_y DB \ln(\frac{D}{y}) \qquad (20)$$

With a hydraulic gradient of unity for the vertical flow through Section PR, the volume of recharge to groundwater table is:

$$V_r = S_y K_u W\ t \qquad (21)$$

Setting Eq 20 equal to Eq 21 yields the recession function of water mound as:

$$y = D\ e^{\frac{-K_u t}{H}} \qquad (22)$$

When the exponent in Eq 22 is small enough, the recession function is reduced to a linear equation as:

$$y = (1 - \frac{K_u t}{H}) \qquad (23)$$

POTENTIAL FLOW MODEL FOR A CIRCULAR BASIN

The flow pattern of infiltrating water below a circular basin is described in Figure 4. Such a three dimensional axially symmetric flow can be described by the stream function as

$$\psi = \pi \frac{f}{D} r^2 y \qquad (24)$$

in which r = radius at distance y. Eq 24 satisfies the continuity and irrotationality equations for a three-dimensional axi-symmetric flow. The streamlines below the basin are distributed as concentric circles with $\psi = 0.0$ along the y-axis and ψ =q at the circumference of the basin bottom, i.e. Point C(r,y) = (Ro ,D) in Figure 4. The infiltration volume rate, q, released from the circular basin is:

$$q = f \pi R_o^{\ 2} \qquad (25)$$

in which R_o = radius of circular basin. To maintain the continuity of flow, the radius, r, of a horizontal cross section at a specified y, can be obtained by Eq 24 as:

$$r = \sqrt{\frac{D}{y}} R_o \qquad (26)$$

For instance, at Section PR in Figure 4, the radius of the area of recharge must pass Point R (R,H) and is equal to:

$$R = \sqrt{\frac{D}{H}} R_o \qquad (27)$$

To repeat the similar approach to an infiltration trench, the required subsurface geometry below a circular basin is described as:

$$\frac{H}{R_o} = \sqrt{\frac{\lambda_r \ln \lambda_y}{2(\lambda_y{}^2 - 1)}} \tag{28}$$

$$\lambda_r = \frac{f}{K_r} \tag{29}$$

in which K_r= hydraulic conductivity in the radial direction. Again, in practice, the value of H is the least one between Eq 28 and the existing thickness of aquifer. Substituting Eq 28 into Eq 7 yields

$$\frac{D}{R_o} = \lambda_y \sqrt{\frac{\lambda_r \ln \lambda_y}{2(\lambda_y{}^2 - 1)}} \tag{30}$$

Similarly, the equivalent homogenous hydraulic conductivity, K_u , is equal to

$$K_u = \sqrt{K_r K_y} \tag{31}$$

During the growing process, the incremental water storage volume in the soil medium is

$$dV = S_y \pi r^2 dy = S_y \pi \frac{D}{y} R_o{}^2 dy \tag{32}$$

The flow rate of recharge at Section PR in Figure 4 is:

$$q_o = S_y K_u \pi R^2 = S_y K_u \pi \frac{D}{H} R_o{}^2 \tag{33}$$

Substituting Eq's 25, 32 and 33 into Eq 14 yields an identical growth function to Eq's 17 through 19. To repeat the similar exercise, an identical recession function to Eq's 22 and 23 can also be achieved for the water mound below a circular basin.

CALIBRATION FOR GROWTH FUNCTION

To predict the growth of a water mound, the most important parameter is the unsaturated hydraulic conductivity, K_u, which is not a constant, but varies with respect to soil moisture content. However, in this study, an attempt to achieve an average or representative value for K_u was conducted by minimizing the difference between the observed and predicted heights of a water mound during its growth. Table 1 presents nine published cases for water mound studies and re-analyses of these nine cases by the potential flow model. For the cases involved a steady state, the saturated depth, D, of the water mound is predicted by Eq 12 for a trench or Eq 30 for a circular basin using a selected K_u based on the agreement with the observed height. For the cases with a known operation time, Eq 17 is used with a selected K_u to match with the observed height. For all cases, the effective thickness of aquifer, H, is the smaller one between the existing thickness and the required depth by Eq's 10 or 28.

In order to derive the relationship among design parameters, a dimensional analysis is performed in this study and results in:

$$\frac{K_u}{K_s} = \phi(P_s) \tag{34}$$

$$P_s = \frac{f}{K_s}\frac{L}{H} \tag{35}$$

in which K_s = saturated hydraulic conductivity, P_s = subsurface geometric parameter, and L = B for a trench basin or L = R_o for a circular basin. In this study, the foregoing nine cases form a data base to calibrate Eq 34. Figure 5 presents the functional relation for Eq 34 and leads to

$$\frac{K_u}{K_s} = 2.97\frac{f}{K_s}\frac{L}{H} \quad (r^2 = 0.91 \text{ for water mound growth process}) \tag{36}$$

in which r2 = correlation coefficient. Eq 36 is empirical and derived for K_u to predict the growth of a water mound. Eq 36 implies that a higher K_u can be developed when a higher infiltration rate is applied to a larger basin with a shallower aquifer. This tendency can be explained by the water retention process in the soil.

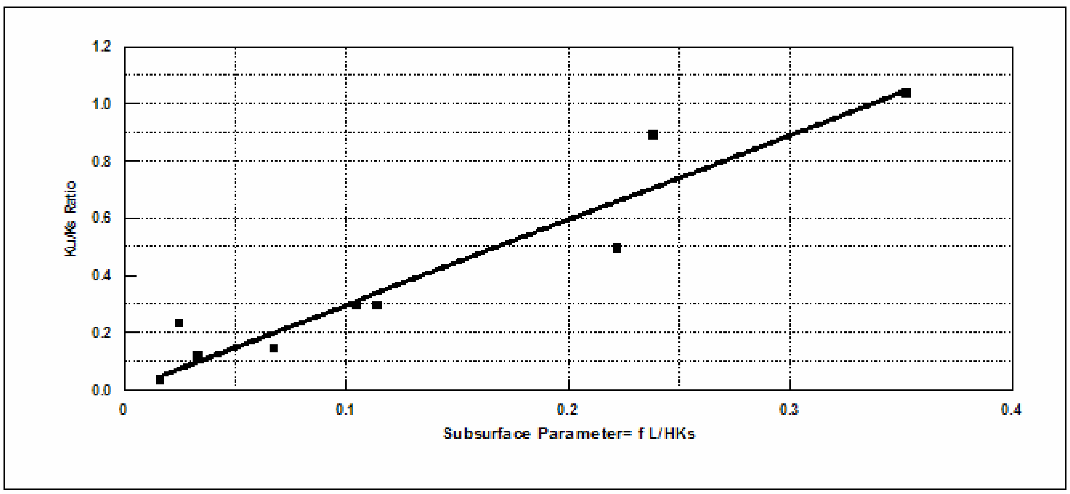

Figure 5 Variation of Soil Hydraulic Conductivity with respect to Soil Saturation

EVALUATION OF RECESSION FUNCTION

During the rest period, the saturated depth, D, begins to recede by recharging the groundwater table at Section PR in Figure 3. For a given time t, Eq 22 predicts the location of the receding water front, and Eq 20 estimates the water volume of recharge which is also equal to the water volume released from the soil medium. Eq 22 essentially depicts the diffusion process of the saturated depth, D, into the effective thickness, H, of the aquifer below the basin by a decay coefficient determined by the hydraulic conductivity. In this study, the potential flow model is applied to four cases reported by Sumner, et el. in 1999. The basin under tests has a radius of 25 meters and is operated under a loading of 0.3 or 1.05 meter per day. The thickness of the aquifer under tests is set to be either 12 or 4 meters.

The receding water front is a process of soil drainage by the gravity. Due to the soil hysteretic behavior, Eq 36 developed for the growth function may not be suitable for the recession function. Therefore, the recession curves for four cases are predicted by the best fitted value of K_u and presented in Table 2. For Cases 1 and 2, the thickness of aquifer is 12 m which is close to the required depth of 12.37 m by Eq 28. The loading of 0.3 m/d is a normal operation, compared to K_s = 2.1 m/d. Both Cases 1 and 2 demonstrate the hysteretic symptom of the soil. However, Cases 3 and 4 demand a similar Ku for both the growth and recession processes of the water mounds. This fact may be caused by the high loading of 1.05 m/d for Case 3 and the shallow aquifer of only 4 meters in Case 4. In this study, the four cases reveal the variations of K_u during the recession, but they are insufficient to lead to any conclusion. In fact, after an intense storm event, the basin may be operated with a high infiltration rate like Case 3. As a result, Eq 36 can be a good estimator for K_u. Figure 6 presents comparisons of predicted recession curves for the first 10 days bewteen the potential flow model and VS2D model (Sumner et. Al. 1999).

CONCLUSIONS

(1) In this study, the potential flow model for the infiltrating flow underneath an infiltration basin has been expanded into the predictions of the growth and recession of a water mound. After evaluating nine observed cases, the unsaturated hydraulic conductivity used in the model has been related to the subsurface geometry parameter as described by Eq 36. Eq 36 is recommended to estimate K_u for the growth of a water mound. It may be suitable to apply Eq 36 to the recession of a water mound under an intense loading or shallow aquifer. Further research is needed to understand the variation of K_u during the recession of a water mound.

(2) The potential flow model is developed for the infiltrating water flow under a short term operation, not for a long term underground water storage operation. It is more applicable to 2 to 5 acre storm water infiltration basin designs. The potential flow model does not require detailed site specifics. It is simple in use and provides reasonable assessments on water mounding impacts. It can be a useful tool at the planning stage when little design information is available, but it does not replace the necessity of more complicated modeling studies to refine the predictions.

APPENDIX I. - REFERENCES

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