Guo, James C.Y. “Roll Waves in High Gradient Channel”, *Journal of IWRA Water International, Vol 24, No. 1, March 1999*

SLUG AND PULSATING FLOW IN HIGH GRADIENT CHANNEL

a revision of the paper published on
Journal of IWRA Water International, Vol 24, No. 1, March 1999.

By James C.Y. Guo, PhD., P.E.
Department of Civil Engineering, University of Colorado at Denver, Denver, Colorado 80204. E-mail:
James.Guo@cudenver.edu.

INTRODUCTION

Supercritical flow occurs in steep channels. A small perturbation in a supercritical flow may be dampened out or amplified to roll waves, depending on the stability of the flow. When the channel is shallow and steep, the surface velocity of the flow may become less than its wave speed. Consequently, the uniform flow as predicted by Manning’s equation breaks into a train of traveling waves or pulses as a response to the temporary force balance among gravitational force, skin friction, and internal turbulent stress. Roll waves in a pulsating flow exhibit similar characteristics to moving oblique jumps. They are the result of transition from a wall boundary layer flow to a turbulent state of flow [1]. After a pulsating flow is developed in a steep channel, roll waves progress downstream and eventually break and form hydraulic bores or shock waves with rough tumbling heads and smooth tails.

Design of high gradient channel is challenging because the interchange between flow depth and velocity head is sensitive to channel shape, curvature, and slope. Often the design velocity in a concrete flood control channel can be as high as 20 to 30 feet/second which is equivalent to velocity head of 6 to 14 ft. One of major concerns in the design of high gradient channel is to make sure that the selected channel cross section can sufficiently accommodate the increase of flow depth due to roll waves. Escoffier and Boyd [2] demonstrated that the ratio of the velocity of the flow to its wave celerity, i.e. Froude number, divides the open channel flow regime into subcritical and supercritical flows. And, the ratio of incremental flow velocity to incremental wave celerity divides the supercritical open channel flow regime into uniform and pulsating flows. Vedernikov’s number [3][4] was also developed to identify the existence of pulsating flow in high gradient channels [5].

Roll waves in a pulsating flow are formed through a continuous growth of disturbances. The complex structure of roll waves consists of a series of bores separated by smooth variable water depth [6]. The heights of roll waves are random and temporal. Several studies [7][8] applied normal probability distribution to describe laboratory and field data of roll waves. However, they are not adequately to relate roll waves to channel shape and slope. Thorsky and Haggman [9] indicated that the channel cross section can have a decisive influence on the generation of roll waves, and channel shapes with no roll waves can be achieved. However such an important design guideline has not been adequately addressed. In 1995, the Corps of Engineers [10] summarized their flood control channel experience in the Los Angeles area and published design curves using Vedernikov’s number developed for rectangular channels as a criterion to identify the pulsating flows in trapezoidal channels. Although these design criteria and curves are useful in alerting the existence of roll waves, none of them provides the needy information of roll wave height for channel designs. Secondly, further derivations of Vedernikov’s number for trapezoidal channels are necessary.

This paper presents revised Vedernikov’s number for trapezoidal channels. A new design chart is presented as an improvement to the current practice. Further, roll waves in this paper are modeled as a moving hydraulic jump. As a result, the height of roll waves can be estimated by design flow, channel shape, slope, and roughness. Design examples demonstrate that the height of roll waves predicted by the moving hydraulic jump is similar to that by the wave frequency distributions. The ratio of flow depth to channel width is identified as an important factor to reduce or eliminate roll waves on a steep slope. Although roll waves can be accommodated by additional freeboard, the optimal channel shapes in terms of the least flow area are often to be those that sustain a supercritical uniform flow, not a pulsating flow.

CRITERIA OF INSTABILITY OF CHANNEL FLOWS

Studies of roll waves were performed primarily in connection with the mechanism of instability of uniform flow on a steep slope. Vedernikov number is often used as a criterion to identify if the uniform flow is stable. Vedernikov number is defined as:

$$Nv = m(1 - R\frac{dP}{dA})F \qquad (1)$$

in which m = 2/3 when Manning's equation is used or ½ when Chezy's equation is used, R = hydraulic radius, P = wetted perimeter, F = Froude number, and A = flow area. To be a stable uniform flow, Nv shall be less than or equal to 1. To apply Eq 1 to a trapezoidal channel, the parameters can be derived as:

$$R = \frac{A}{P} = \frac{y(b+zy)}{b+2ky} \qquad (2)$$

$$k = \sqrt{1+z^2} \qquad (3)$$

$$\frac{dP}{dA} = \frac{2k}{b+2zy} \qquad (4)$$

in which y = flow depth, b= channel bottom width, and z = channel side slope. Normalizing Eq's 2 through 4 by the channel bottom width and then substituting Eq's 2,3 and 4 into Eq 1 yields:

$$F \le \frac{3}{2}\left[\frac{(1+2kY^*)(1+2zY^*)}{1+2zY^* + 2kzY^{*2}}\right] \qquad (5)$$

$$Y^* = \frac{y}{b} \qquad (6)$$

Eq 5 is the limiting Froude number for having stable uniform flows in high gradient trapezoidal channels. When z =0, it reduces to

$$F \le \frac{3}{2}(2Y^* + 1) \qquad (7)$$

Eq 7 agrees with the straight line on Plate B-7 of Corpse of Engineers' Hydraulic Design of Flood Control Channels [10] when the channel side slope is zero. But it was also found that Eq 7 was misused to produce similar design curves for trapezoidal channels. Figure 1 presents the revised limiting Froude numbers for channels with z = 0.0, 0.5, 1.0, 2.0 and 3.0. These curves separate the supercritical flow regime into stable uniform flows with Froude numbers equal to or less than the limiting Froude number and pulsating flows with Froude numbers greater than the limiting Froude number. In general, deep and narrow channels tend to carry stable uniform flow and shallow and wide channels tend to carry pulsating flows. When the flow Froude number <1.5, a supercritical uniform flow can be sustained in trapezoidal channels, and when the flow Froude number >3.0, roll waves in pulsating flows are expected.

Care has to be taken when designing a steep channel. Selections of *y/b* ratio, channel slope, and roughness shall follow the above criteria to avoid roll waves. Otherwise mitigation has to be provided, including additional freeboard or rougher linings.

HEIGHTS OF ROLL WAVES

Development of roll waves is a continuous amplification of a small perturbation [6]. Block [7] recommended the required distances for the growth of roll waves to various stages. For the purpose of design, this complicated process can be approximated by the model of positive surges which have an advancing front with the profile similar to a moving hydraulic jump. When the height of the surge is small, the surge appears undular like an undular jump. When the height is increasing, the undulation will eventually disappear and the surge will have a sharp and steep front. The moving hydraulic jump was investigated by Liggett [5] to solve the gradually varied flow equation, and achieved the same criteria of instability as Vedernikov's number. This conclusion further confirms the dynamic similarity between moving hydraulic jumps and roll waves.

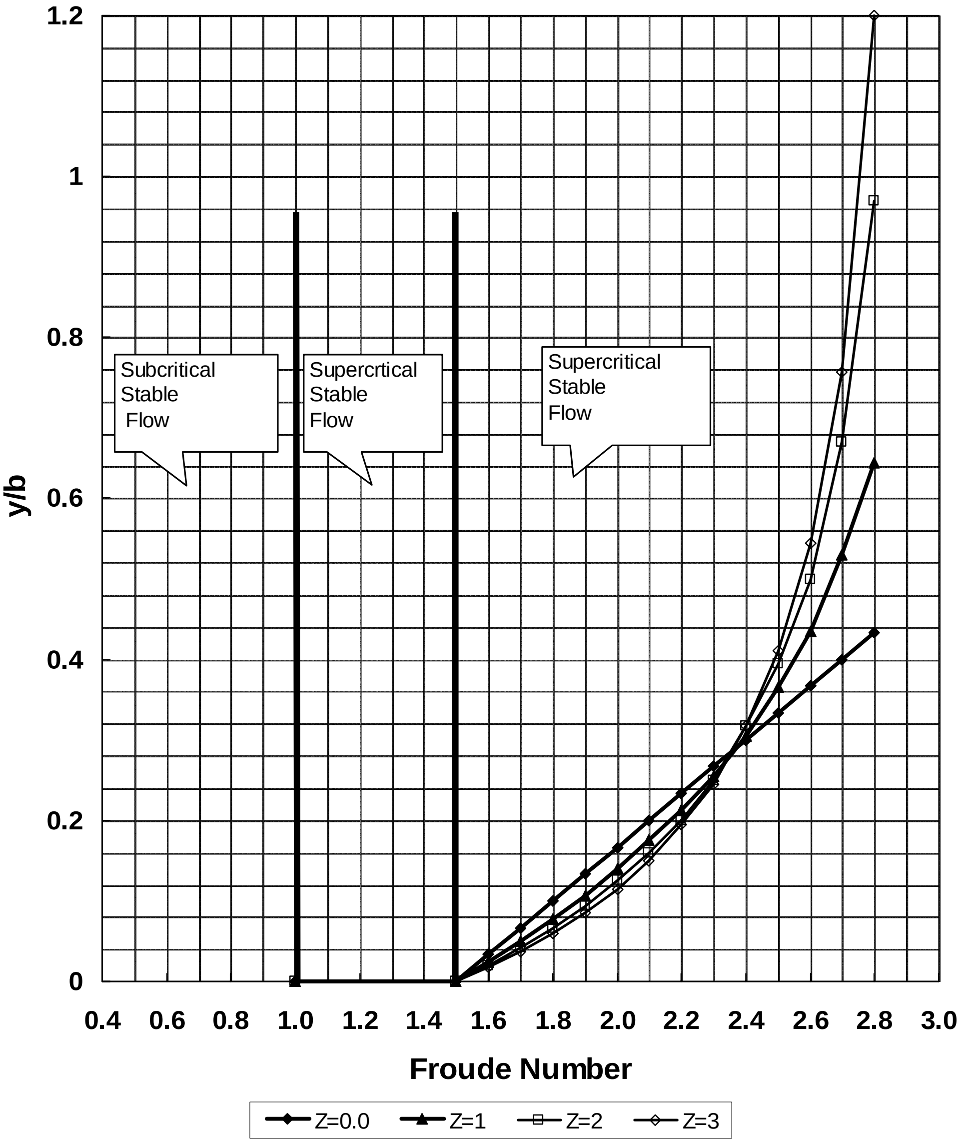

Figure 1 Regimes for Open Channel Flows

In this study, the roll wave front is considered as a positive surge. The unsteady flow pattern can be converted to its steady pattern by a coordinate system moving with the bore velocity. As illustrated in Figure 2, the continuity principle is:

$$(V_2 - V_w) A_2 = (V_1 - V_w) A_1 \tag{8}$$

Re-arranging Eq 8 yields:

$$V_2 = \frac{(V_1 - V_w)A_1 + V_w A_2}{A_2} \tag{9}$$

And the momentum principle is:

$$(V_w - V_2)(V_2 - V_1) = (\overline{y_2} - \frac{A_1}{A_2}\overline{y_1})g \tag{10}$$

in which V_w = wave velocity, V = flow velocity, A = flow area, g = gravitational acceleration, and $\overline{y}$ = distance to the centroid of the flow area, approximated by 0.5y.

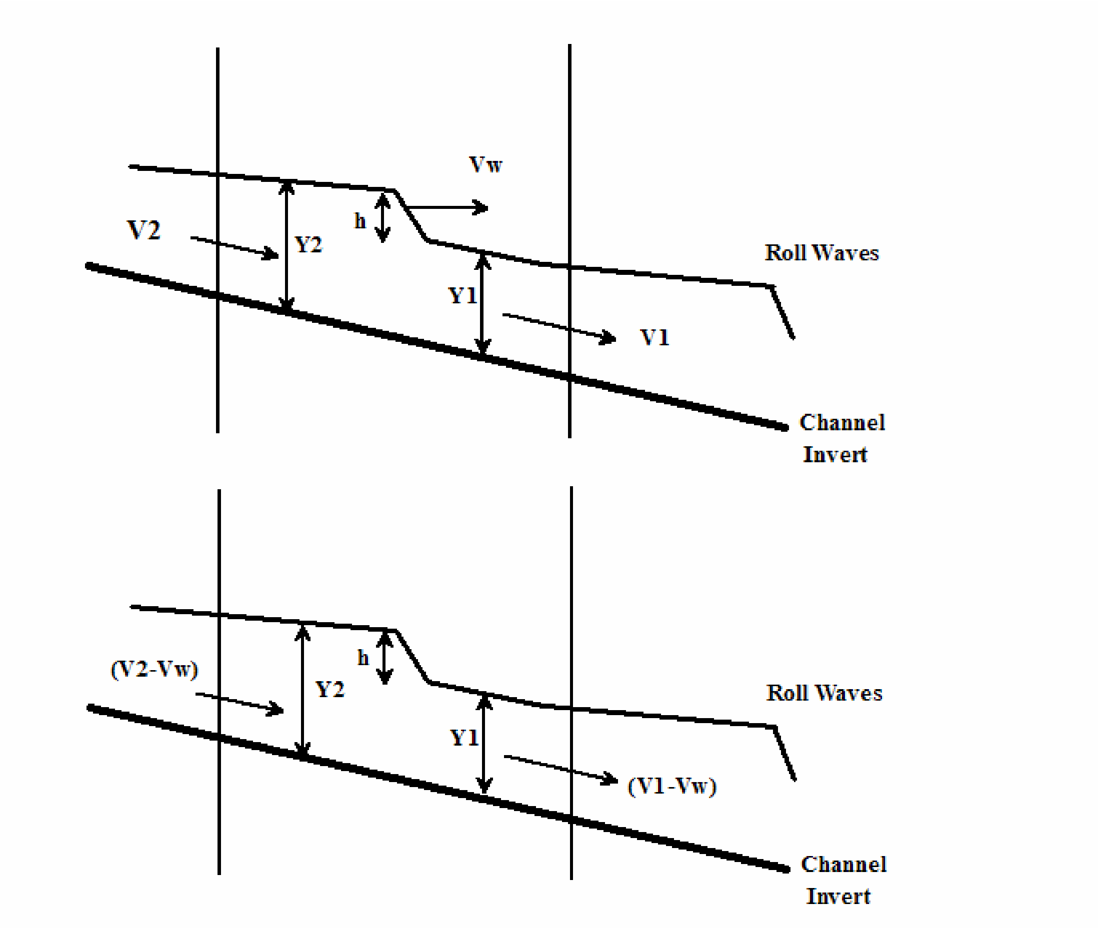

Figure 2 Illustration of Moving Hydraulic Jump

The subscript 2 denotes the design condition including roll waves, and the subscript 1 represents the limiting condition defined by Vedernikov number. Solving Eq's 8, 9, and 10 simultaneously yields

$$V_w = V_1 + \sqrt{\frac{(A_2\overline{y_2} - A_1\overline{y_1})g}{A_1(1 - A_1/A_2)}} \tag{11}$$

Let the height of roll waves be the difference between flow depths as

$$h = y_2 - y_1 \qquad (12)$$

$$C = V_w - V_2 \qquad (13)$$

in which h = roll wave height, and C = wave celerity. In a high gradient channel, the water surface in a cross section is fluctuating due to cross waves [8]. Considering that the roll waves near the center of the channel section is similar to that in a rectangular channel, the representative height of roll waves can be derived by simplifying Eq's 8 and 9 for rectangular channel. Details can be found elsewhere [1]. Canceling the wave velocity in Eq's 8 and 9, the height of roll waves is derived as:

$$h = \frac{C^2}{g}\left(\frac{2y_1}{y_1 + y_2}\right)\left(\frac{V_2}{C} - \frac{V_1}{C}\right) = \frac{C^2}{g}\left(\frac{2y_1}{y_1 + y_2}\right)(F_2 - F_1) \qquad (14)$$

Eq 14 is only applicable if $F_2 > F_1$. Otherwise h=0.

$$F_1 = \frac{V_1}{C} \qquad (15)$$

$$F_2 = \frac{V_2}{C} \qquad (16)$$

In practice, F_2 is the Froude number for the design discharge and F_1 is the limiting Froude number determined by Vedernikov's number. When the height of roll waves is small compared with the depth of flow, i.e. $y_1 \approx y_2$, Eq 14 is reduced to

$$h = \frac{C^2}{g}\left(\frac{V_1}{C} - \frac{V_2}{C}\right) = \frac{(V_w - V_2)^2}{g}(F_1 - F_2) \qquad (17)$$

Eq 17 agrees with Chow's two-dimensional surge model [1] and can provide an estimated wave height of the spectrum of roll waves. To apply the above procedure to the design of a high gradient channel, the design condition, at first, shall be evaluated by Eq 5 or Figure 1. If the design condition exceeds the limiting condition, a pulsating flow is expected. The height of roll waves can be estimated by Eq 17 for the freeboard design.

DESIGN EXAMPLES

Like oblique jumps, roll waves are formed through a successive amplification. Methods derived to quantify heights of roll waves, while recognizing the uncertainty and scarcity of data, are to provide estimates from a statistical point of view [7]. For instance, the example of a 9.8-ft wide rectangular concrete channel was used by French [11] to illustrate how to use the wave frequency distributions to estimate the height of roll waves. The maximum height of roll waves in this channel is estimated to be 1.8 ft when the discharge is 320 cfs on a 10% slope. The same example is employed to test the moving hydraulic jump model by the following procedures:

(1) Check the Stability of the Design Flow

The uniform flow for the example has a depth of 1.0 ft. Since the design Froude number is 5.66 which is greater than the limiting Froude number of 1.8 determined by Eq 7 or Figure 1, roll waves are expected.

(2) Calculation of the Height of Roll Waves

2.1 Guess the height of roll waves, h in Eq 12. Solve for y_2 and V_2 by Eq 12 and Eq 9 respectively. Solve for V_w by Eq 11 and C by Eq 13. Calculate the height of roll waves by Eq 17.

2.2 Revise the value of h based on the calculated value and repeat Steps 2.1 and 2.2 until the convergence is achieved between the guessed and calculated height of roll waves.

This procedure leads to a prediction of roll waves of 1.73 feet for the aforementioned case, which is comparable to the method of wave frequency distributions.

Further, a sensitivity investigation on roll waves was also tested for a discharge of 5000 cfs on a 3.0% concrete slope. Channel shapes considered are the combinations among bottom widths of 10-, 15-, and 20-ft, and side slopes of 0, 1, 2, and 3 ft/ft. As shown in Table 1, the wider the channel is, the lower the roll waves are. The steeper the side slope is, the higher the roll waves are. And, the shallower the channel is, the higher the roll waves are. Therefore, there exists a tradeoff between channel width and depth when roll waves are taken into account. Under the consideration of roll waves, the minimal flow area is achieved by a 15-ft rectangular channel for this case. In fact, a 15-ft rectangular channel is the one that sustains a supercritical uniform flow without roll waves. Similar conclusions were also observed for many other cases.

Channel Width b ft	Side Slope z ft/ft	Limiting Froude Number	Uniform Flow Depth y/b	Design Flow Froude Number	Roll Wave Height h ft	Wave Speed fps	Water Flow Depth ft	Water Flow Area sq ft
10	0	5.03	1.175	2.19	0.00	62.02	11.76	117.60
10	1	2.81	0.665	3.66	3.32	65.45	9.97	199.10
10	2	2.82	0.564	3.83	3.01	61.04	8.65	236.15
10	3	2.86	0.510	3.84	2.57	56.87	7.67	253.19
15	0	3.00	0.500	2.86	0.00	60.05	7.51	112.65
15	1	2.51	0.369	3.73	3.95	65.03	9.48	232.04
15	2	2.61	0.328	3.84	3.26	60.31	8.18	256.74
15	3	2.71	0.304	3.83	2.72	56.28	7.29	268.55
20	0	2.36	0.287	3.21	3.90	64.03	9.63	192.60
20	1	2.26	0.237	3.76	4.22	63.67	8.97	259.79
20	2	2.42	0.219	3.83	3.42	59.26	7.79	277.17
20	3	2.54	0.206	3.82	2.84	55.50	6.96	284.77
25	0	2.07	0.190	3.40	4.73	63.45	9.49	237.25
25	1	2.09	0.167	3.78	4.27	61.86	8.45	282.76
25	2	2.25	0.158	3.82	3.49	57.99	7.43	296.02
25	3	2.39	0.151	3.81	2.92	54.60	6.69	301.19

Table 1 Case Study for Channel Geometries for Stable Supercritical Flow

CONCLUSION

To design a high gradient channel, the design condition must be evaluated by Vedermikov's number as a stability criterion. In this study, Vedernikov's number was expanded from rectangular channels to trapezoidal channels. The widely used design curves recommended by the Corps of Engineers for flood channel designs was revised to identify the existence of pulsating flow in high gradient channels. When Froude number is less than 1.5, flows in trapezoidal channels are stable, and when Froude number is greater than 3.0, roll waves are expected in high gradient trapezoidal channels. Figure 1 is produced to determine the stability of flow when Froude number is between 1.5 and 3.0. When a design condition exceeds the limiting condition, the increased flow depth due to roll waves shall be accommodated by the additional freeboard.

Roll waves in pulsating flows are further modeled by a moving hydraulic jump. Design procedures developed in this paper provide estimations of wave height in the same magnitude as solutions obtained from the wave frequency distributions. However, the frequency distributions were only developed for three ranges of Froude number between 3.45 to 5.60. The moving hydraulic jump model developed in this study provides estimations of roll wave heights as Froude number is greater than 1.50.

REFERENCES

1. Chow.V. T., “Open Channel Hydraulics”, McGraw-Hill Book Company, New York, 1959

2. Escoffier, F. F., and Boyd, M. B., “Stability Aspects of Flow in Open Channels”, J. of the Hydraulics Division, ASCE, Vol 88, No HY 6, November, 1962

3. Vedernikov, V. V., “Conditions at the Front of a Translatory Wave Distributing a Steady Motion of a Real Fluid”, C.R. USSR Acad. Sci., Vol 48, No 4, 1945.

4. Vedernikov, V.V., “Characteristic Features of a Liquid Flow in an Open Channel”, C. R. USSR Acad. Sci., Vol 52, 1946.

5. Liggett, James A. “Stability”, Chapter 6, Unsteady Flow in Open Channels, Edited by Mahmood, K., and Yevjevich, V., Vol 1, Water Resources Publications, Littleton, Colorado, 1975.

6. Mayer, P.G.W. “ A study of Roll Waves and Slug Flows in Inclined Open Channels”, PhD Dissertation, Cornell University, 1957

7. Brock, R.R., “Development of Roll Wave Trains in Open Channels,” Proceedings of the ASCE J of the Hydraulics Division, Vol 95, No HY 4, July, 1969.

8. Stonstreet, Scott E., “Stochsatic Determination of Wave Heights for Flood Control Channels”, ASCE Conference Proceedings of North American Water and Environment Congress, 1996.

9. Thorsky, G. N., and Haggman, D.C., Discussion of “Development of Roll Wave Trains in Open Channels,” ASCE J of Hydraulic Division, Vol 96, No HY 4, April, 1970.

10. US Army Corps of Engineers, “Hydraulic Design of Flood Control Channels,” Engineer Manual No. 1110-2-1601, Department of the Army, Washington D.C., 20314-1000, 1991.

11. French, Richard H. “Open Channel Hydraulics”, McGraw Hill Company, New York, New York, 1985